Model Exam-IV

MATHEMATICS

XII STD CBSE

Time = 3hrs Max Marks = 100

General Instructions

1. All questions are compulsory.

2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

Section-A

1. Find X, if Y = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and 2X+Y = $\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

2. If a,b,c are in A.P, then the determinant $\begin{bmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{bmatrix}$ is - - - - -

3. Find the inverse of $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

4. If $y = Ae^{5x}+Be^{-5x}$, then prove that $y_2-25y = 0$

5. Prove that the logarithmic function is strictly increasing on $(0,\infty)$

6. Evaluate $\int e^x(\tan x + \sec^2 x)dx$

7. Find the integrating factor of the differential equation $(1-y^2)\frac{dx}{dy} + yx = ay; \quad -1 < y < 1$

8. If $\vec{a}+\vec{b} = 3\vec{i}+2\vec{j}-\vec{k};$ *and* $\vec{a}-\vec{b} = 3\vec{i}+4\vec{j}-3\vec{k}$, find the magnitude and the direction cosines of $2\vec{a}-3\vec{b}$

9. In a box containing 100bulbs, 10 are defective. Find the probability that out of a sample of 5 bulbs, none is defective.

10. With usual notation find p for B.D. if n = 6 and if P(x = 4) = P(x = 2).

Section-B

11. Consider the binary operation $\wedge$ on the set {1,2,3,4,5, 6} defined by $a \wedge b$ = max{a,b}. Write the operation table of the operation $\wedge$. Find the identity element if any.

12. In a legislative assembly election, a political group hired a public relations firm to promote its candidate in three ways, telephone, house calls and letters. The cost per contact (in paise) is given in matrix A as $A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \text{telephone} \\ \text{Housecall} \\ \text{Letter} \end{matrix}$ (Costpercontact). The number of contact of each type made in two cities X and Y is given by $B = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10{,}000 \end{bmatrix} \begin{matrix} X \\ Y \end{matrix}$ (Tele House Letter) Find the total amount spent by the group in the two cities X and Y.

13. Verify mean value theorem for $f(x) = x^3 - 5x^2 - 3x$ in the interval [a,b] where a = 1 and b = 3. Find all $c \in (1,3)$ for which $f'(c) = 0$

14. If $y = \left(\tan^{-1} x\right)^2$, Prove that $(1+x^2)^2 y_2 + 2x(1+x^2) y_1 = 2$.

15. (a) Find the approximate value of f(3.02) where $f(x) = 3x^2+5x+3$. (OR)
(b) Prove that the curves $y^2 = x$ and $xy=k$ cut orthogonally if $8k^2 = 1$

16. Evaluate $\int \frac{dx}{3x^2+13x-10}$

17. Evaluate $\int \sqrt{x^2 - a^2}\, dx$

18. Evaluate $\int_0^{\frac{p}{2}} \left(\frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} \right) dx$

19. Solve $\sin^{-1}\left(\frac{dy}{dx}\right) = (x+y)$

20. (a) If $\vec{d}_1$ and $\vec{d}_2$ are the diagonals of a parallelogram with sides $\vec{a}_1$ and $\vec{a}_2$. Find the area of the parallelogram in terms of $\vec{d}_1$ and $\vec{d}_2$, and hence find the area with $\vec{d}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{d}_2 = 3\hat{i} - 2\hat{j} + \hat{k}$

(or)

(b) If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angle between them, then prove that $\sin\frac{q}{2} = \frac{1}{2}\left|\hat{a} - \hat{b}\right|$

21. A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once.

22. (a) Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

(OR)

(b)Find the probability distribution of number of doublets in three throws of a pair of dice.

Section-C

23. Solve $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{p}{4}$,

24. (a) Find the inverse of the matrix $A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$ and show that $aA^{-1} = (a^2 + bc + 1)I - aA$.

(OR)

(b) If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.Verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}

25. Sand is pouring from a pipe at the rate of $12m^3$/sec.The falling sand forms a cone on the ground in such a way that the height of the cone is always 1/6 th of the base. How fast is the height of the sand cone is increasing, when height is 4.

26. Find the area lying above x-axis and included between the circle $x^2+y^2=8x$ and the parabola $y^2 = 4x$.

27. Find the equation of the plane which contains the line of intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$ and perpendicular to the plane $5x+3y+6z+8=0$.

28. Find the vector equation of the line passing through the point (1,2,–4) and perpendicular to the two lines: $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

29. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and I unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 unit/kg of vitamin C. It costs Rs.50 per kg to purchase Food I and Rs. 70 per kg to purchase Food II. Formulate this problem as a linear programming problem to minimize the cost of such a mixture.

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