Time: 3 Hours **Guess Paper** **Max. Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six mark each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION A

1. Let * be a binary operation defined by a * b =2a.b-7. Is * associative?
2. If $y = \cos(a)$, find $\frac{da}{dy}$.
3. Which type of matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is this?
4. If A is a square matrix of order n, then adj (adj A) is equal to
   (a) $|A|^{n-1} A$ (b) $|A^{n} A$ (c) $|A|^{n-2} A$
5. Find the abscissa for which, the tangent to the curve $y = x^2 - 5$ is parallel to the line $y = -x - 6$.
6. If $\hat{i}, \hat{j}$ *and* $\hat{k}$ are unit vectors along X,Y and Z axis respectively, find the value of $\hat{i}.(\hat{j}x\hat{k}) + \hat{j}.(\hat{i}x\hat{k}) + \hat{k}.(\hat{i}x\hat{j})$
7. If Cartesian equation of the line AB is $\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$, find the direction ratios of a line parallel to AB.
8. Determine order and degree of differential equations: $y = x \cdot \frac{dy}{dx} + a \cdot \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.
9. Find the angle between $\vec{a}$ and $\vec{b}$, *such that* $|\vec{a}| = 2, |\vec{b}| = \sqrt{3}$ *and* $|\vec{a} \times \vec{b}| = 3$.
10. Find the adjoint of a matrix $\begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$

SECTION B

11. Prove that the relation R on the set N × N defined by
(a, b) R (c, d) ⇔ a + d = b + c for all
(a, b), (c, d) ∈ N × N is an equivalence relation.

12. Prove that: $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}=\frac{\pi}{4}$

13. Using properties of determinants prove that

$$\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$$

14. If f(x) , defined by the following, is continuous at x=4, find the values of a and b.

$$f(x)=\begin{cases} \frac{x-4}{|x-4|}+a & if\ \ x<4 \\ a+b & if\ \ x=4 \\ \frac{x-4}{|x-4|}+b & if\ \ x>4 \end{cases}$$

15. If $x^p y^q = (x + y)^{p+q}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

16. Evaluate: $\int\frac{\sin 2x}{\sin 5x\sin 3x}dx$. Or $\int\frac{(x-3)e^x}{(x-1)^3}dx$

17. Verify Lagrange's mean value theorem for the function f(x) = x + $\frac{1}{x}$ in [1,3].

18. Evaluate: $\int_0^{\pi/2}\frac{x}{\sin x+\cos x}dx$

19. If $x=a(\theta-\sin\theta), y=a(1+\cos\theta)$ find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{2}$.

20. Find the equation of the line through the point (-1, 2, 3) which is perpendicular to the lines $\frac{x}{2}=\frac{y-1}{-3}=\frac{z+2}{-2}$ and $\frac{x+3}{-1}=\frac{y+2}{2}=\frac{z-1}{3}$.

21. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3,\ |\vec{b}|=4,\ |\vec{c}|=5$ and each one of them being perpendicular to the sum of other two, Find $|\vec{a}+\vec{b}+\vec{c}|$.

22. A bag contains 6white and 3 black balls. Another bag B contains 4 white and 5 black balls. A ball is transferred from bag A to the bag B and a ball is taken out of the second bag.Find the probability of this ball being black.

SECTION C

23. A window is in form of a rectangle surmounted by a semi-circle. If the perimeter of the window is 100m, find the dimensions of the window so that maximum light enters through the window.

24. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Or

Given that $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ Find AB. Use this to solve the following system of linear equations:

$$\begin{aligned} x - y + z &= 4 \\ x - 2y - 2z &= 9 \\ 2x + y + 3z &= 1 \end{aligned}$$

25. Find the area of the following region: $\{(x, y):\ y^2 \le 4x,\ 4x^2 + 4y^2 \le 9\}$.

26. Solve $(x + 2y^2)\dfrac{dy}{dx} = y$, given that x = 2 when y = 1.

27. For what value of 'a' are the four points A (3, 2, 1) B (4, a, 5), C (4, 2,-2) and D (6, 5,-1) coplanar?

28. From a well shuffled pack of 52 cards. 3 cards are draw one-by-one with replacement. Find the probability distribution of number of queens.

29. A company manufactures, two types of toys-A and B. Toy A require 4 minutes for cutting and 8 minutes for assembling and Toy B requires 8 minutes for assembling. There are 3 hours and 20 minutes available in a day for cutting and 4 hours for assemble. The profit on a piece of toy A is Rs. 50 and that on toy B is Rs. 60. How many toys of each type should be made daily to have maximum profit? Solve the problem graphically.

Or

An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

Great is the art of beginning, but greater is the art of ending.