RELATION AND FUNCTIONS

1. Let T be the set of all triangles in a plane with R a relation in T given by R = $\{(T_1,T_2):T_1 \cong T_2\}$. Show that R is an equivalence relation.
2. Show that the relation R in the set A=$\{1,2,3,4,5\}$ given by R=$\{(a,b):|a\text{-}b| \text{ is even}\}$ is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other and all the elements of $\{2,4\}$ related to each other. But no element of $\{1,3,5\}$ is not related to any element of $\{2,4\}$.

3. Let L be the set of all lines in XY plane and R be the relation in L defined as R=$\{(L_1,L_2):L_1 \parallel L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4 .

4. Show that the signum function $f:R \to R$, given by $f(x)=\begin{cases} |x| & ,x \neq 0 \\ 0 & , x=0 \end{cases}$ is neither one-one and onto.

5. Show that $f:N \to N$, given by $f(x)=\begin{cases} x+1 & ,x \text{ is odd} \\ x-1 & , x \text{ is even} \end{cases}$ is one-one and onto.

6. For a non empty set X , consider the binary operation $*: P(X) \times P(X) \to P(X)$ given by $A* B=A \cap B \ \forall \ A,B \in P(X)$ where P(X) is the power set . Also , show that X is the identity element for this operation and X is the only invertible element in P(X) w.r.t. the operation * .

7. Let A = N x N and * be the binary operation on A given by (a , b) * (c, d) = (a+c , b+d). Show that * is commutative and associative . Find the identity element for * in A , if any.

8. Consider $f:R_+ \to [-5,\alpha)$ given by $f(x) = 9x^2 + 6x-5$. Show that f is invertible with $f^{-1}(y) = \dfrac{\sqrt{y+6}-1}{3}$

8. If $f(x) = \dfrac{4x+3}{6x-4}$, $x \neq \dfrac{2}{3}$, show that $fof(x) = x$. What is the inverse of f ?

10. Let $f:x \to y$ and $g:y \to z$ be two invertible functions. Then prove that gof is

invertible with $(gof)^{-1} = f^{-1}og^{-1}$.

11. Let $f:N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f:N \rightarrow S$ where S is the range of f , is invertible . Find the inverse of f .

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