Sample paper -2010
Class : XII
Sub : **MATHEMATICS**

Time allowed: 3 hrs M.Marks:100

*General Instructions***:**

(i) All questions are compulsory.
(ii) The question paper consists of 29 questions divided into three sections – A, B and C. Section A comprises of 10 questions of 1 mark each; Section B comprises of 12 questions of 4 marks each and Section C comprises of 7 questions of 6 marks each.
(iii) Use of calculator is not permitted. You may ask for logarithmic tables if required.
(iv) **Draw all figures by pencil only.**

SECTION : A

1. Prove that $f : R \to R$ given by $f(x) : x^3 + 1$ is one-one.

2. Evaluate : $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

3. For what value of k , the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse .

4. Evaluate : $\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

5. Find the equation of line joining the points $(1,2)$ and $(3,6)$ using determinants .

6. Evaluate : $\int e^{e^x} e^x dx$

7. Evaluate : $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} x^2 \sin x dx$

8. Find the co-ordinate of the foot of perpendicular drawn from origin to the plane $x + y + z = 1$.

9. Find the vector in the direction of $\vec{r} = \hat{i} + 2\hat{j} - 3\hat{k}$ whose magnitude is 7 .

10. Find the area of parallelogram having diagonals $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$.

Contd.

SECTION : B

11. Consider $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Find the inverse of f.

12. Using properties of determinant prove that $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

13. Prove that the function $f(x) = \begin{cases} \dfrac{x}{|x| + 2x^2} &, x \neq 0 \\ k &, x = 0 \end{cases}$ is discontinuous at x = 0 regardless the value of k

14. Prove that : $\dfrac{9\pi}{8} - \dfrac{9}{4}\sin^{-1}\dfrac{1}{3} = \dfrac{9}{4}\sin^{-1}\dfrac{2\sqrt{2}}{3}$

15. If $y = ae^{mx} + be^{nx}$, prove that $\dfrac{d^2y}{dx^2} - (m+n)\dfrac{dy}{dx} + mny = 0$

16. Prove that the curves $x = y^2$ and xy = k cut at right angles if $8k^2 = 1$.

17. Evaluate : $\displaystyle\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

18. The volume of the spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of the balloon after t seconds.

19. Show that the differential equation $2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ is homogeneous and find its particular solution , given that x = 0 when y = 1 .

20. Prove by vector method that in triangle ABC , $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ where $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AB} = \vec{c}$.

OR,

If $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.

21. Find the equation of the plane through the intersection of the planes x + y + z = 1 and 2x + 3y + 4z =5 which is perpendicular to the plane x – y + z = 0 .

OR,

Prove that if a plane has the intercepts a , b , c and is at a distance **p** units from origin , then $a^{-2} + b^{-2} + c^{-2} = p^{-2}$.

22. A card from a pack of 52 cards is lost . From the remaining cards of the pack , two cards are drawn and are found to be both diamonds . Find the probability of the lost card being a diamond .

Contd.

SECTION : C

23. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the following system of equations

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

24. If a line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube then prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

25. Show that the volume of the greatest right circular cylinder that can be inscribed in a cone oh height h and semi vertical angle α is $\frac{4}{27}\pi h^3 \tan^2\alpha$

26. Find the area bounded by the region $\left\{(x,y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1 \le \frac{x}{a} + \frac{y}{b}\right\}$

OR,

Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0, x = 4, y = 4$ and $y = 0$ into three equal parts .

27. Evaluate : (i) $\int_1^4 (|x-1| + |x-2| + |x-3|)dx$

(ii) $\int \frac{dx}{x^4 - 1}$

28. A dietician wishes to mix two types of foods in such a way that vitamin contents of the food mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 unit/kg vitamin A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg vitamin A and 2 unit/kg of vitamin C. It cost Rs.50 per kg to purchase food I and Rs.70 per kg to purchase food II. Formulate this problem as a LPP to minimize the cost of such a mixture

29. A fair coin is tossed 10 times , find the probability of

(i) exactly six heads
(ii) at least six heads
(iii) at most six heads

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