Q-1 Find x, if $\begin{pmatrix} 5 & 3x \\ 2y & z \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 12 & 6 \end{pmatrix}^T$.

Q-2 For what value of a, $\begin{pmatrix} 2a & -1 \\ -8 & 3 \end{pmatrix}$ is a singular matrix?

Q-3 A square matrix A, of order 3, has |A| = 5, find |A.adjA|.

Q-4 Using properties of determinants, prove that :

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = (1+a^2+b^2+c^2).$$

Q-5 If $A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}$, find A^{-1} and hence solve the following system of equations :

$$2x+y+3z = 3$$
$$4x-y = 3$$
$$-7x + 2y + z = 2$$

OR

Using elementary transformations, find the inverse of the matrix :

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$$

Q-6 What is the value of $|3\ I_3|$, where I_3 is the identity matrix of order 3?

Q-7 For what value of k, the matrix $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible?

Q-8 If A is a matrix of order 2x3 and B is a matrix of order 3x5, what is the order of matrix (AB)′ or T?

Q-9 If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2-5A-14I = 0$. Hence find A^{-1}.

Q-10 Using properties of determinants, show that

$$\Delta = \begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+b)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc\ (a+b+c)^3$$

Ans:1 $x = 4$

Ans:2 $a = \frac{4}{3}$

Ans:3 $$\text{LHS} = \frac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ ac^2 & bc^2 & c(c^2+1) \end{vmatrix} = \frac{abc}{abc}\begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$$R_1 \to R_1+R_2+R_3 = \begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$$|1 \quad 1 \quad 1| \qquad\qquad |1 \quad 0 \quad 0|$$

Ans:4 $|A| = 2(-1) - 1(4) + 3(1) = -3 \neq 0 \quad A^{-1} = \frac{1}{|A|} \text{adj } A$

The cofactors are

$$A_{11} = -1,\ A_{12} = -4,\ A_{13} = 1$$
$$A_{21} = 5,\ A_{22} = 23,\ A_{23} = -11$$
$$A_{31} = 3,\ A_{32} = 12,\ A_{33} = -6$$

$$\therefore A^{-1} = -\frac{1}{3}\begin{pmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{pmatrix}$$

Ans:5 Given equations can be written as

$$\begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \text{ or } A \cdot X = B$$

$$\therefore X = A^{-1} \cdot B$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -6 \\ -27 \\ 14 \end{pmatrix}$$

$$\therefore x = -6,\ y = -27,\ z = 14$$

OR

$$\text{Let } A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} \text{ then } \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$$R_3 \to R_3 - 3R_1 \Rightarrow \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 0 & +1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} A$$

$$R_2 \leftrightarrow R_3 \Rightarrow \begin{pmatrix}1 & -1 & 2\\0 & 1 & -2\\0 & 2 & -3\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\-3 & 0 & 1\\0 & 1 & 0\end{pmatrix} A$$

$$R_3 \to R_3\text{-}2R_2 \Rightarrow \begin{pmatrix}1 & -1 & 2\\0 & 1 & -2\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\-3 & 0 & 1\\6 & 1 & -2\end{pmatrix} A$$

$$R_1 \to R_1 + R_2 \Rightarrow \begin{pmatrix}1 & 0 & 0\\0 & 1 & -2\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}-2 & 0 & 1\\-3 & 0 & 1\\6 & 1 & -2\end{pmatrix} A$$

$$R_2 \to R_2 + 2R_3 \Rightarrow \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{pmatrix} A$$

Hence $A^{-1} = \begin{pmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{pmatrix}$

Ans:6 27

Ans:7 17

Ans:8 5x2

Ans:9 $A = \begin{bmatrix}3 & -5\\-4 & 2\end{bmatrix} \Rightarrow A^2 = \begin{bmatrix}3 & -5\\-4 & 2\end{bmatrix}\begin{bmatrix}3 & -5\\-4 & 2\end{bmatrix} = \begin{bmatrix}29 & -25\\-20 & 24\end{bmatrix}$

$$A^2 - 5A - 14I = \begin{bmatrix}29 & -25\\-20 & 24\end{bmatrix} -5\begin{bmatrix}3 & -5\\-4 & 2\end{bmatrix} -14\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}29 & -25\\-20 & 24\end{bmatrix} + \begin{bmatrix}-15 & 25\\20 & -10\end{bmatrix} + \begin{bmatrix}-14 & 0\\0 & -14\end{bmatrix}$$

$$\begin{bmatrix}29\text{-}15\text{-}14 & -25\text{+}25\text{-}0\\-20\text{+}20\text{+}0 & 24\text{-}10\text{-}14\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} = 0$$

Premultiplying $A^2\text{-}5A\text{-}14I = 0$ by A^{-1}, we get

$A^{-1}.\ A^2 - 5A^{-1}A - 14A^{-1}I = 0$

or, $A\text{-}5I - 14A^{-1} = 0$

$$\text{or } A^{-1} = \frac{1}{14}\ (A\text{-}5I) = \frac{1}{14}\left\{\begin{bmatrix}3 & -5\\-4 & 2\end{bmatrix} + \begin{bmatrix}-5 & 0\\0 & -5\end{bmatrix}\right\}$$

$$= \frac{1}{14}\begin{bmatrix}-2 & -5\\-4 & -3\end{bmatrix}$$

Ans:10

$$\Delta = \begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix}$$

Operating $R_1 \to aR_1,\ R_2 \to bR_2,\ R_3 \to cR_3$, to get

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(b+c)^2 & a^2b & a^2c \\ ab^2 & b(a+c)^2 & b^2c \\ ac^2 & b^2c & c(a+b)^2 \end{vmatrix} = \frac{abc}{abc}\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

Operating $c_2 \to c_2 - c_1,\ c_3 \to c_3 - c_1$, to get

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (a+c)^2-b^2 & 0 \\ c^2 & 0 & (a+b)^2-c^2 \end{vmatrix} = (a+b+c)^2\begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & a+b-c & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

Operating $R_1 \to R_1 - (R_2 + R_3)$ to get

$$\Delta = (a+b+c)^2\begin{vmatrix} 2bc & -2c & -2b \\ b^2 & a+c-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix} \quad c_2 \to c_2 + \frac{1}{b}c_1,\ c_3 \to c_3 + \frac{1}{c}c_1$$

$$= (a+b+c)^2\begin{vmatrix} 2bc & 0 & 0 \\ b^2 & a+c & \frac{b^2}{c} \\ c^2 & \frac{c^2}{b} & a+b \end{vmatrix}$$

$$(a+b+c)^2\left[2bc(a^2+ac+ab+bc-bc)\right] = (a+b+c)^2(2bc)\,a(a+b+c)$$

$$= (a+b+c)^3 \cdot 2abc$$