Year of Quality education 18th

Code No. **Series AG-Q** CLASS XII **TMG -D/79/89**

- Please check that this question paper contains 3 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.

General Instructions: -

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 marks each, Section B is of 12 questions of 4 marks each and Section C is of 7 questions of 6 marks each.
3. Write the serial number of the question before attempting it.
4. If you wish to answer any question already answered, cancel the previous answer.
5. In questions where internal choices is provided. You must attempt only one choice.

MATHEMATICS

Time Allowed : 3 hours **Maximum Marks : 100**

PART – A

1. Let $f: R \to R$ be defined by $f(x) = x^2 - 3x + 1$, Find $f[f(x)]$.
2. Find the value of $\cos[\sec^{-1} x + \operatorname{cosec}^{-1} x]$.
3. If $\vec{AB} = 4\hat{i} + 5\hat{j} - 3\hat{k}$ and the position vector of point B is $3\hat{i} - \hat{j} + 2\hat{k}$, find the position vector of point A.
4. Evaluate: $\int_0^a \frac{1}{x + \sqrt{a^2 - x^2}} dx.$
5. Find the value of k the system of equation has non – trivial solution x-2y + 3z =0 ; 3x –y + 2z = 0 & 2x + k y + z = 0 .
6. Find the value of $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$.
7. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric then find the value of x .
8. Determine the integrating factor of differential equations: $(x - \sin y)dy + \tan y dx = 0, y(0) = 0.$
9. The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5 . Find the probability that the problem will be solved.
10. Find the equations of planes parallel to the plane $x - 2y + 2z = 3$ which are at a unit distance from the point (1, 2, 3).

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PART – B

11. Prove that : $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$.

12. Find a unit vector perpendicular to the plane ABC where A,B and C are the points(3,- 1 ,2), (1,-1,-3) and (4,-3,1) respectively.

13. Let A {2, 3, 4, 5, 6, 7, 8, 9 } .Let R be the relation on A defined by $\{(x,y): x \in A \,\&\, x$ divides y)}. Find (i) R (ii)Domain of R (iii) Range of R (iv) R^{-1} state whether or not R^{-1}, is (a) Reflexive (b) symmetric (c) transitive .

OR

let $A = N \times N$ and * be an binary operation defined by $(a,b)*(c,d)=(ac,bd)\forall a,b,c,d \in N$ on the set R , then (i) Prove that * is a binary operation on N (ii) Is * commutative ? (iii)Is associative (iv) Find the identity element for * on $N \times N$ if any .

14. Examine the continuity of the function f defined by f(x) = $\begin{cases} x\sin\frac{1}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$ at x = 0.

15. Prove that : $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$.

16. If $y = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$ then prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 4$.

17. Evaluate: $\int_{-\pi/4}^{\pi/4} \frac{x+\pi/4}{2-\cos 2x}dx$.

18. Obtain the differential equation by eliminating a and b from the equation $y = e^x(a\cos x + b\sin x)$.

OR

Prove that $y^2 = a(b^2 - x^2)$ is a solution of differential equation $xy\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$.

19. By examining the chest X-ray, the probability that T.B. is detected when a person is actually suffering is 0.99.The probability that the doctor diagnoses is incorrectly that a person has T.B. on the basis of x-ray is 0.001.In a certain city , 1in 1000 persons suffers from TB. A person is selected at random and is diagnosed to have T.B. What is the chance that he actually has T.B.?

OR

Two persons A & B throw a pair of dice alternately beginning with A .Find the probability that B gets a doublet and wins before A gets a total of 9 to win.

20. Evaluate : $\int_0^4 e^{2-3x}dx$. as limit of a sum.

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21.Find the points at which the function f given by $f(x)=(x-2)^4(x+1)^3$ has (a) local maxima (b) local minima (c) point of inflexion

OR

A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most .Its semi vertical angle is $\tan^{-1}(1/2)$. Water is poured into it at a constant rate of 5 cubic meter per minute .Find the rate at which the level of the water is rising at the instant when the depth of the water in the tank is 10 cm .

22.A company has estimated that the probabilities of success for three products introduced in the market are $\frac{1}{3}, \frac{2}{5}$ & $\frac{2}{3}$ respectively. Assuming independence, find (i) the probability that the three products are successful (ii) the probability that none of the products is successful.

PART – C

23.Reduce the equation of plane in symmetrical form; the equation of line are x–y+2z = 5, 3x + y + z = 6.

OR

Find the vector equation of the following plane in scalar product form $\vec{r}=(i-j)+\lambda(i+j+k)+\mu(i-2j+3k)$.

24.A line makes angles $\alpha,\beta,\gamma and \delta$ with the diagonals of a cube, prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$.

25.Find the area of the region bounded by the curves $y=x^2+2, y=x, x=0$ & $x=3$.

OR

Find the area of the region $\{(x,y): x^2\le y\le |x|\}$.

26.Evaluate: $\int_0^{\pi/2} x\cot x\,dx=\frac{\pi}{2}\log 2$.

27.A factory produces two kinds of toys ; Dolls and Cars. Machines A and B are used for not more than 1½ hours each to process Dolls and Cars. On machine A each Car is processed for 1 minute and each Doll for 2 minutes. On machine B each Car is processed for 2 minutes and each Doll for 1 minute. The profit on each Car is Rs 3 and on each Doll is Re 1. How many of each kind of toy should be produced in the factory to give the maximum profit ?

28.Find A^{-1}, where $A=\begin{bmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{bmatrix}$ .Hence solve the system of equation $x+2y-3z=-4; 3x-3y-4z=11; 2x+3y+2z=2$.

29.A window has the shape of a rectangle surmounted by an equilateral triangle If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.

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