



Code No. **Series AG-4**

CLASS XII

TMG-D/79/89

- Please check that this question paper contains 3 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.

General Instructions: -

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 marks each, Section B is of 12 questions of 4 marks each and Section C is of 7 questions of 6 marks each.
3. Write the serial number of the question before attempting it.
4. If you wish to answer any question already answered, cancel the previous answer.
5. In questions where internal choices is provided. You must attempt only one choice.

MATHEMATICS

Time Allowed : 3 hours

Maximum Marks : 100

PART – A

1. A four digit number is formed using the digit 1,2,3,5 with no repetitions . Find the probability that the number is divisible by 5.
2. On expanding by first row, the value of a third order determinant is $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$. Write the expression for its value on expanding by 2nd column. Where A_{ij} is the cofactor of element a_{ij}
3. If $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$, find $x, 0 < x < \frac{\pi}{2}$ when $A + A' = I$
4. Find λ when the projection of $\hat{i} + \lambda\hat{j} + \hat{k}$ on $\hat{i} + \hat{j}$ is $\sqrt{2}$ units.
5. Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.
6. If B is a skew symmetric matrix, write whether the matrix (ABA') is symmetric or skew symmetric.
7. Find the order and degree of the differential equation, $x^2 \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^3 + 5 = 0$.
8. The Cartesian equation of a line are $6x - 2 = 3y + 1 = 2z - 2$. Find the direction ratio of the line .
9. Evaluate $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$.

10. Evaluate : $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

PART – B

11. Show that either $a + b + c = 0$ or $a = b = c$ If $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$.

12. Show that the functions $f(x) = |x+2|$ is a continuous at every $x \in R$ but fails to be differentiable at $x = -2$.

OR

Let $f(x) = \begin{cases} \frac{1 - \sin^3 x}{\cos^2 x} & \text{if } x < \frac{\pi}{2} \\ a & \text{if } x = \frac{\pi}{2} \\ b(1 - \sin x) & \text{if } x > \frac{\pi}{2} \end{cases}$ If $f(x)$ is continuous at $x = \frac{\pi}{2}$, find a and b .

13. If $x = a \sin pt$ and $y = b \cos pt$, find the value of $\frac{d^2y}{dx^2}$ at $t = 0$.

14. Prove that the function $f : R - \{3\} \rightarrow R - \{1\}$ given by $f(x) = \frac{x-2}{x-3}$ is bijection.

15. Show that $\int_0^a \sin^{-1} \sqrt{\frac{x}{a+x}} dx = \frac{a}{2}(\pi - 2)$

16. Find the equation of tangent and normal to the curve: $x = a \cos t + at \sin t$; $y = a \sin t - at \cos t$, at any point 't'. Also prove that the normal to the curve is at a constant distance from the origin.

OR

Find all the local maximum values and local minimum values of the function

$f(x) = \sin 2x - x, -\frac{\pi}{2} < x < \frac{\pi}{2}$.

17. Evaluate : $\int \frac{2 + \sin x}{1 + \cos x} e^{x/2} dx$

18. Find the equation of the plane containing the lines, $\vec{r} = i + j + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and

$\vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$. Find the distance of this plane from origin and also from the point (1,1,1).

OR

Find the distance between the points with position vectors $-\hat{i} - 5\hat{j} - 10\hat{k}$ and the points of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ with the plane $x - y + z = 5$.

19. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of a ΔABC respectively. Find an expansion for the area of ΔABC and hence deduce the condition for the points A, B, C to be collinear.

20. Solve the differential equation : $\frac{d^2x}{dy^2} = 1 + \sin y$ given that $dx / dy = 0$ and $y = 0$ when $x = 0$.

OR

Obtain the differential equation by eliminating a and b from the equation $y = e^x (a \cos x + b \sin x)$.

21. Find $\frac{dy}{dx}$, if $y = \tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right], 0 < |x| < 1$

22. A, B, C shoot to hit a target. If A hits the target 4 times in 7 trails, B hits it 3 times in 5 trials and C hits it 2 times in 3 trials, what is the probability that the target is hit by at least 2 persons ?

PART – C

23. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

24. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.

OR

Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6r\sqrt{3}$.

25. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appears on the two dice. Find the mean or expectation of X.

26. Find the distance between the point P (6,5,9) and the plane determined by points A(3,-1,2), B (5,2,4) and C (-1,-1,6).

27. Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $x^2 = 6y$.

OR

Draw a rough sketch of $y^2 = x+1$ and $y^2 = -x+1$ and determine the area enclosed by the two curves.

28. There are three urns having the following compositions of black and white balls : urn 1 contain 7 white and 3 black balls, urn 2 contain 4 white and 6 black balls & urn 3 contain 2 white and 8 black balls. One of these urns is chosen with probabilities 0.2, 0.6 and 0.2 respectively. From the chosen urn, two balls are drawn at random without replacement. Both the balls happen to be white. Calculate the probability that the balls drawn were from urn III.

29. A dietician mixes together two kinds of food in such a way that the mixture contains at least 6 units of vitamin A, 7 unit of vitamin B, 11 units of vitamin C and 9 units of vitamin D. The vitamin contents of 1 kg of food X and 1 kg of food Y are given below : One kg of food X costs Rs. 5, whereas one kg of food Y costs Rs. 8. Find the least cost of the mixture which will produce the desired diet.

	Vitamin A	Vitamin B	Vitamin C	Vitamin D
Food X	1	1	1	2
Food Y	2	1	3	1