Er Manish Bhadoria's Interactions Study Circle

Strong Foundation for a Bright Future

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Mathematics

(Answers and Solutions to Sample Paper - I)

This sheet contains answers and solutions to the Mathematics sample paper – 01 published on www.cbseguess.com on 25 – 01 – 2010. Please note that many problems are solved only in short.

Answers

1. Prime factorisation of q must be of the form 2^n5^m, where n and m are non-negative integers.
2. Only 1 zero.
3. 9 : 1
4. ± 4
5. $^1/_2$
6. n = 25
7. 3 : 1 : 2
8. 60°
9. $^1/_{26}$
10. Median class is (10 – 15)
11. x = 3, y = 2
12. – 2
13. 31 square units
14. ---
15. (i) $^1/_2$, (ii) $^5/_6$

    Choice

    (i) $^1/_{11}$, (ii) $^2/_3$
16. ---
17. (i) $-^7/_4$, (ii) $-^{115}/_{32}$
18. (3, 4), (6, 2) and (0, 2)
19. AP is 2, 7, 12, 17, ……

    Sum of first fifteen terms = 555
20. ---
21. 3 : 4
22. (-7, 0), area = 53 sq. units
23. ---
24. 5.7 cm
25. 42 cm^2

    Choice

    38.28 cm^2
26. 27 years and 5 years

    Choice

    42 km/h
27. ---

    Choice

    Distance = $10\sqrt{3}$ m,

    Height of other tower = 10 m
28. ---
29. 720 cm^2
30. Mean = 38.8

    Median = 38.57

    Mode = 39

Hints / Solutions

1. ---

2. The graph intersects the x-axis at only one point.

3. $\Delta ABC \sim \Delta ADE$

$$\frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{AB^2}{AD^2} = \frac{3^2}{1^2} = \frac{9}{1}$$ *Ans*

4. For equal roots:

D = 0

$b^2 - 4ac = 0$

$k^2 - 4.1.4 = 0$

$k^2 = 16$

k = ± 4 *Ans*

5. tan A = 1

∴ A = 45°

$$\sin A \cos A = \sin 45°.\cos 45° = \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} = \frac{1}{2}$$ *Ans*

6. $a_n = a + (n-1)d = 141$ (given)

$-3 + (n-1)6 = 141$

After solving: n = 25 *Ans*

7. Let radius of each = r (same bases given)

height of hemisphere is equal to its radius

∴ height of each = r (same heights given)

Vol of cylinder : Vol of cone : Vol of hemisphere

$$\pi(\text{radius})^2(\text{height}) : \frac{1}{3}\pi(\text{radius})^2(\text{height}) : \frac{2}{3}\pi(\text{radius})^3$$

$$\pi r^3 : \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3$$

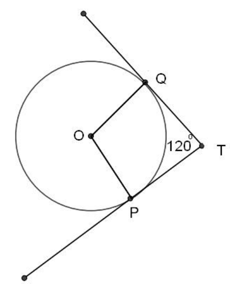
3 : 1 : 2 *Ans*

8. ∠PTQ = 120°

quad. POQT is a cyclic quadrilateral.

∴ ∠POQ + ∠PTQ = 180°

∠POQ = 180° - 120° = 60° *Ans*



9. There are two red queens in a pack (diamond and hearts)

P(red queens) = $\frac{2}{52} = \frac{1}{26}$ *Ans*

10.

class	frequency	Cumulative frequency
0-5	5	5
5-10	9	14
10-15	12	26
15-20	18	44
20-25	6	50

n = 50, $\frac{n}{2} = 25$

median class = class whose cumulative frequency is greater than and nearest to $\frac{n}{2}$.

$\therefore$ median class is (10 – 15) *Ans*

11. Let $\frac{1}{y} = z$

Then equations are:

$4x + 6z = 15$

$6x - 8z = 14$

Solving these, we get $x = 3$, $z = \frac{1}{2}$

$y = \frac{1}{z} = 2$

$\therefore$ x = 3, y = 2 *Ans*

12. ---

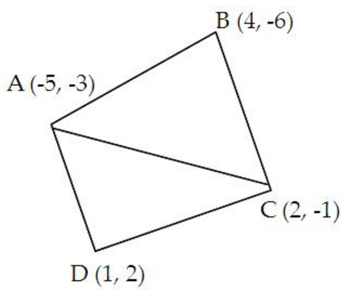
13. ar(quad. ABCD) = ar(ΔABC) + ar(ΔADC)

using formula for area of triangle:

$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

ar(ΔABC) = $\frac{39}{2}$ sq. units

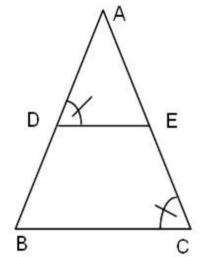
ar(ΔADC) = $\frac{23}{2}$ sq. units



$\therefore$ ar(quad. ABCD) = $\frac{39}{2} + \frac{23}{2}$

$= \frac{62}{2} = 31$ sq. units *Ans*

14. $\frac{AD}{DB} = \frac{AE}{EC}$



$\therefore$ DE ∥ BC (converse of BPT)

∠ADE = ∠ABC (corresponding angles)

But ∠ADE = ∠ACB (given)

$\therefore$ ∠ABC = ∠ACB

$\therefore$ AB = AC (sides opposite to equal angles of a triangle are equal)

So, ΔABC is an isosceles triangle. *Proved*

15. (i) prime numbers on a die: 2, 3, 5 (total three)

P (prime number) = $\frac{3}{6} = \frac{1}{2}$ *Ans*

(ii) numbers less than 6: 1, 2, 3, 4, 5 (total 5)

P (number < 6) = $\frac{5}{6}$ *Ans*

Choice

(i) cards removed: 13 diamonds, 4 queens, 4 jacks.

Queen and Jack of diamond are common in these.

$\therefore$ total cards removed

= 13 + 4 + 4 – 2 = 19

Remaining cards = 52 – 19 = 33

Face cards in a pack: 4 Kings + 4 Queens + 4 Jacks = total 12

Removed = 4 Queens, 4 Jacks and 1 King (of diamond)

$\therefore$ face cards remain = 12 – 9 = 3

P (face card) = $\frac{3}{33} = \frac{1}{11}$ *Ans*

(ii) black cards removed = 4 (two black Queens and two black Jacks)

$\therefore$ black cards remain=26 – 4 = 22

P (black card) = $\frac{22}{33} = \frac{2}{3}$ *Ans*

16. ---

17. $\alpha + \beta = \frac{5}{2}$, $\alpha\beta = 4$

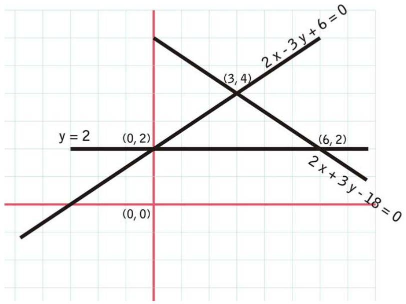
(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{5}{2}\right)^2 - 2 \times 4 = \frac{25}{4} - 8 = -\frac{7}{4}$ *Ans*

(ii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$

$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$= -\frac{115}{32}$ *Ans*

18. Vertices of the triangle are (3, 4), (6, 2), and (0, 2).



19. Let the AP be a, a+d, a+2d, ……

$a_8 = a + 7d = 37$ ………..(i)

$a_{15} = 15 + a_{12}$

a + 14d = 15 + a + 11d

solving, d = 5

then, a + 7d = a + 7×5 = 37

⇒ a = 2

∴ AP is 2, 7, 12, 17, …… *Ans*

$S_{15} = \frac{15}{2}[2\times2 + (15-1)5] = 555$ *Ans*

20. LHS = $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = \frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{\sec^2\theta - 1}} = \frac{2\sec\theta}{\tan\theta}$,

$= \frac{2/\cos\theta}{\sin\theta/\cos\theta} = \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta$ = RHS

Choice

To prove: $\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$

or, to prove: $\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} - \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta} = 2$

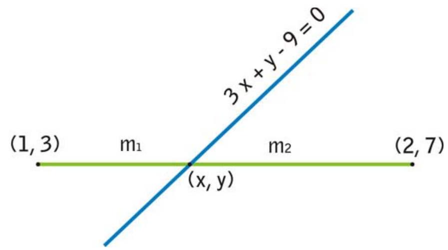
LHS

$= \sin\theta\left[\frac{\cot\theta - \operatorname{cosec}\theta - \cot\theta - \operatorname{cosec}\theta}{\cot^2\theta - \operatorname{cosec}^2\theta}\right]$

$= \sin\theta\left[\frac{-2\operatorname{cosec}\theta}{-1}\right]$

$= 2\sin\theta\operatorname{cosec}\theta = 2 = \text{RHS}$

21. Let the line 3x + y – 9 = 0 divides the line segment joining the points (1, 3) and (2, 7) in the ratio $m_1:m_2$ at point (x, y).



Then by section formula,

$x = \frac{2m_1 + m_2}{m_1 + m_2}$, $y = \frac{7m_1 + 3m_2}{m_1 + m_2}$

point (x, y) lies on line 3x+y–9=0.

So, it will satisfy the equation.

$3\left[\frac{2m_1 + m_2}{m_1 + m_2}\right] + \left[\frac{7m_1 + 3m_2}{m_1 + m_2}\right] - 9 = 0$

Solving, $m_1 : m_2 = 3 : 4$ *Ans*

22. Let that point on *x* axis be (x, 0).

Then, $\sqrt{(x-2)^2 + (0+5)^2} = \sqrt{(x+2)^2 + (0-9)^2}$

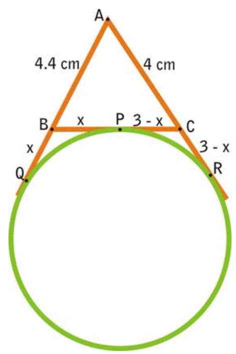
Solving, x = - 7 ⇒ point is (-7, 0).

Area of triangle can be found by the formula used in question 13.

Δ = 53 sq. units *Ans*

23. ---

24. Lengths of tangents drawn from an external point to a circle are equal.



∴ AQ = AR

AB + BQ = AC + CR

4.4 + x = 4 + 3 – x

x = 1.3 cm

so, AQ = 4.4 + 1.3 = 5.7 cm *Ans*

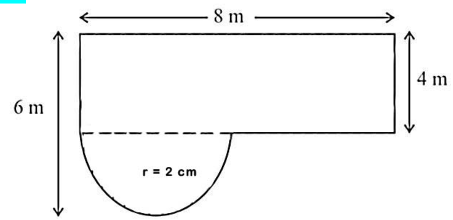
25. Area of shaded region = area of square – area of four quadrants of circle

$= (14)^2 - 4 \times \frac{1}{4}\pi (7)^2 = 42 \text{ cm}^2$ *Ans*

Choice

Total area = area of rectangle + area of semicircle

$= (8 \times 4) + \frac{1}{2}\pi (2)^2 = 38.28 \text{ m}^2$ *Ans*



26. Let Asha (Mother) is x years old and Nisha (Daughter) is y years old presently.

Then, according to the problem -

$x = y^2 + 2$ ……(i)

Difference in their ages = $(x - y)$ years.

So, Daughter will grow to her mother's present age after $(x - y)$ years.

At that time: Mother's age = $x + (x - y) = 2x - y$ and Daughter's age = x

Now, According to the problem-

$2x - y = 10y - 1$

Substituting for x from eqn. (i)

$2(y^2 + 2) - 11y + 1 = 0$

$2y^2 - 11y + 1 = 0$

On solving this quadratic equation, we get $y = 5$ and $y = \frac{1}{2}$

$y = \frac{1}{2}$ is not acceptable because ages are in years.

$\therefore$ $y = 5$ and $x = y^2 + 2 = 27$

So, Asha's age = 27 years and Nisha's age = 5 years. *Ans*

Choice

Time = $\frac{\text{distance}}{\text{speed}}$

Let the usual speed of train be x km/h

For first part of journey: distance = 63 km, speed = x km/h

$\therefore$ time taken = $\frac{63}{x}$ h ……(i)

For second part: distance = 72 km, speed = $(x + 6)$ km/h

$\therefore$ time taken = $\frac{72}{x+6}$ h ……(ii)

According to the problem -

Total time taken = 3 h

$\frac{63}{x} + \frac{72}{x+6} = 3$

This simplifies into quadratic equation: $x^2 - 39x - 126 = 0$.

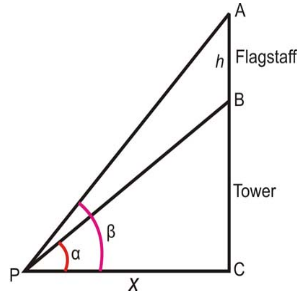
on solving, $x = 42$ and $x = -3$ km/h.

speed cannot be negative,

therefore original speed of the train is 42 km/h. *Ans*

27. Let AB be the flagstaff and BC be the tower.

Also let distance between point and tower be x.



In ΔBCP, $\tan\alpha = \frac{BC}{x}$

Or, $x = \frac{BC}{\tan\alpha}$ ……(i)

In ΔACP, $\tan\beta = \frac{AC}{x} = \frac{AB+BC}{x} = \frac{h+BC}{x}$

Or, $h + BC = x\tan\beta$

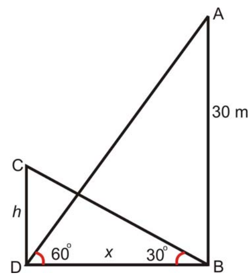
So, $h = \frac{BC}{\tan\alpha}\tan\beta - BC = \frac{BC\tan\beta - BC\tan\alpha}{\tan\alpha}$

$h\tan\alpha = BC(\tan\beta - \tan\alpha)$

$\therefore BC = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$ *Proved*

Choice

Let AB and CD be the towers and the distance between the towers be x m.



In ΔABD, $\tan 60° = \frac{AB}{BD} = \frac{30}{x}$

$\Rightarrow \sqrt{3} = \frac{30}{x}$, $x = \frac{30}{\sqrt{3}} = 10\sqrt{3}$

$\therefore$ distance between towers = $10\sqrt{3}$ m *Ans*

In ΔCDB, tanCDB, $\tan 30° = \frac{CD}{DB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} = \frac{h}{10\sqrt{3}} \Rightarrow h = 10$ m

Thus, height of the other tower = 10 m *Ans*

28. $AB^2 = AE^2 + BE^2$

$= AE^2 + (BD - DE)^2$

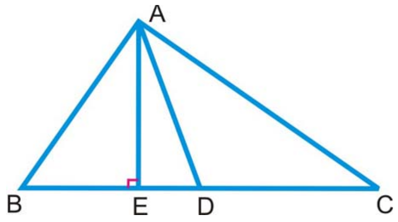
$= AE^2 + (CD - DE)^2$

$= AE^2 + CD^2 + DE^2 - 2\,CD.DE$

$= AD^2 + CD^2 - BC.DE$

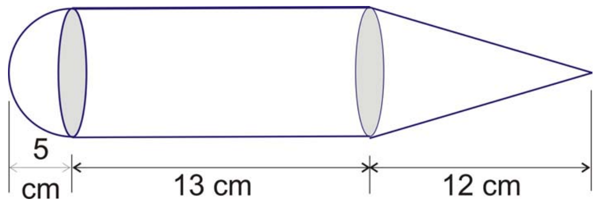
$= AD^2 + (\frac{1}{2}BC)^2 - BC.DE$

$= AD^2 - BC.DE + \frac{1}{4}BC^2$ *Proved*



29. Radius of each part = 5 cm

Slant height of cone, $l = \sqrt{h^2 + r^2} = \sqrt{(12)^2 + (5)^2} = 13$ cm



CSA of toy = CSA of hemisphere + CSA of cylinder + CSA of cone

$= 2\,\pi\,(5)^2 + 2\,\pi\,(5)\,(13) + \pi\,(5)\,(13)$

$= \pi\,[50 + 130 + 65] = \frac{22}{7}[245] = 720\text{ cm}^2$ *Ans*

30.

Class	Class Mark (x_i)	Frequency (f_i)	Cumulative Frequency	f_ix_i
5 – 15	10	2	2	20
15 – 25	20	3	5	60
25 – 35	30	5	10	150
35 – 45	40	7	17	280
45 – 55	50	4	21	200
55 – 65	60	2	23	120
65 – 75	70	2	25	140
		$\sum f_i = 25$		$\sum f_ix_i = 270$

(i) Mean, $\overline{X} = \frac{\sum f_ix_i}{\sum f_i} = \frac{970}{25} = 38.8$ *Ans*

(ii) Finding Median: $\frac{n}{2} = \frac{25}{2} = 12.5 \Rightarrow$ median class is (35 – 45)

Median $= l + \frac{\frac{n}{2} - cf}{F} \times h = 35 + \frac{12.5 - 10}{7} \times 10 = 38.57$ *Ans*

(iii) Finding Mode:

Highest frequency is of class (35 – 45) $\Rightarrow$ modal class is (35 – 45)

Mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 35 + \frac{7 - 5}{14 - 5 - 4} \times 10 = 39$ Ans

----------------------------------- x ----------------------------------