

Roll No.

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MATHEMATICS
SET - B

Time allowed : 3 hr

Maximum Marks : 100

General Instructions:

- (i) All Questions are compulsory.
- (ii) The question paper consists of 29 questions divided into three sections A, B, and C. Section A comprises of 10 questions of **one mark** each. Section B comprises of 12 questions of **four marks** each and Section C comprises of 7 questions of **six marks** each.
- (iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question..
- (iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

Section A

- 1 If $f(x)$ is an invertible function, find the inverse of $f(x) = \frac{3x-2}{5}$.
- 2 Find the principal value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) + \cot^{-1}\left(\cot\frac{7\pi}{6}\right)$.
- 3 If A is non-singular square matrix such that $|A| = 10$, find $|A^{-1}|$.
- 4 If $\text{adj } A = \begin{bmatrix} 3 & 5 \\ 7 & -2 \end{bmatrix}$ and $\text{adj } B = \begin{bmatrix} 2 & -3 \\ -5 & 2 \end{bmatrix}$, find $\text{adj } AB$.
- 5 Find the values of x and y if: $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$
- 6 Find the value of $\int_0^1 x(1-x)^2 dx$
- 7 Write the value of $\int_0^2 x[x] dx$
- 8 Find a vector in the direction of $\vec{a} = 4\hat{i} + \hat{j} + 3\hat{k}$, whose magnitude is 3.
- 9 If $|\vec{a}| = 1, |\vec{b}| = 1, \vec{a} \cdot \vec{b} = \cos\theta$, find $|\vec{a} - \vec{b}|$
- 10 Find k for which the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{6-z}{5}$ are \perp to each other

Section B

- 11 Let $f: W \rightarrow W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f . Here, W is the set of all whole numbers. **OR**
Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$. Is f is one-one and onto? Justify your answer.
- 12 Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$
- 13 If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$. Hence, find A^{-1}
- 14 Show that $f(x) = |x - 2|$, $x \in \mathbb{R}$ is continuous but not differentiable at $x = 2$. **OR**
If $x^m y^n = (x + y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$
- 15 Find the interval in which the function $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is (i) increasing (ii) decreasing

- 16 The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base?

Or

Using differential, find the approximate value of $(0.009)^{1/3}$.

- 17 By using the properties of definite integrals, evaluate: $\int_0^{\pi/4} \log(1 + \tan x) dx$

18 Solve: $\left(x \cos \frac{y}{x} + y \sin \frac{y}{x}\right) y dx = \left(y \sin \frac{y}{x} - x \cos \frac{y}{x}\right) x dy$

- 19 Solve the following differential equation: $(1+y+x^2y)dx + (x+x^3)dy=0$, where $y = 0$ when $x=1$

- 20 If $\vec{a}, \vec{b}, \vec{c}$ are respectively the position vectors of the vertices A, B, C of ΔABC , prove that area of the triangle ABC is given by $\Delta = \frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

- 21 Find the equation of the plane which contains of the line of intersection of the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$ and $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is \perp to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$

- 22 A can hit a target 3 times in 6 shots, B: 2 times in 6 shots and C: 4 times in 4 shots. They fix a volley. What is the probability that at least 2 shots hit ?

OR

How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80% ?

SECTION C

23 Show that: $\Delta = \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$

- 24 Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

OR

Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point (1,4),

25 Evaluate: $\int \frac{1-\sqrt{x}}{1+\sqrt{x}} dx$ OR Show that: $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x}) dx = \sqrt{2}\pi$

- 26 Using integration, find the area of the region $\{(x, y) : |x-1| \leq y \leq \sqrt{5-x^2}\}$

- 27 Find the equation of the plane which contains the two parallel line $\frac{x-3}{3} = \frac{y+4}{2} = \frac{z-1}{1}$ and

$$\frac{x+1}{3} = \frac{y-2}{2} = \frac{z}{1}$$

- 28 An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that
(i) all will bear 'X' mark. (ii) not more than 2 will bear 'Y' mark
(iii) at least one ball will bear 'Y' mark
(iv) the number of balls with 'X' mark and 'Y' mark will be equal

- 29 A dietician wishes to mix together two kind of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of 1 kg food is given below

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs. 16 and one kg of food Y costs Rs. 20. Find the least cost of the mixture which will produce the required diet?

Best of Luck

Answers (MOCK TEST -2)

Section A

- 1 $f^{-1}(y) = \frac{5y+2}{3}$ 2 $\frac{\pi}{3}$ 3 $\frac{1}{10}$ (4) $\begin{bmatrix} -15 & 16 \\ -1 & -29 \end{bmatrix}$ 5 $x=2$ and $y=-8$
(6) $\frac{1}{12}$ 7 $\frac{3}{2}$ (8) $\frac{3}{\sqrt{26}}(4\hat{i}-\hat{j}+3\hat{k})$ 9 $2 \sin \frac{\theta}{2}$ (10) $-\frac{10}{7}$

Section B

- 13 $\frac{1}{14} \begin{bmatrix} -2 & -5 \\ -4 & -3 \end{bmatrix}$ 15 Increasing in $(-\infty, -1)$ and $(1, \infty)$ and decreasing in the interval $(-1, 1)$
16 decreasing at the rate $\sqrt{3} \text{ b cm}^2/\text{s}$ or 0.2083 17 $\frac{\pi}{8} \log 2$ 18 $xy \cos \frac{y}{x} = C$
19 $\Rightarrow yx + \tan^{-1}x = \frac{\pi}{4}$ 21 $\vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) - 41 = 0$ 22 $\frac{2}{3}$ or 3 times

Section C

- 24 the point $(2, 2)$ on the curve $y^2 = 2x$ is at a minimum distance from the point $(1, 4)$
25 $-2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x}\sqrt{1-x} + C$ 26 $\frac{5}{2} \left[\sin^{-1} \left(\frac{2}{\sqrt{5}} \right) + \sin^{-1} \left(\frac{1}{\sqrt{5}} \right) \right] - \frac{1}{2}$
27 $8x + y - 26z + 6 = 0$ 28 (i) $\left(\frac{2}{5}\right)^6$ (ii) $7\left(\frac{2}{5}\right)^4$ (iii) $1 - \left(\frac{2}{5}\right)^6$ (iv) $20\left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3$
29 if 2 kg of food X and 4 kg of food Y are used. The minimum cost C will be Rs 112