Roll No.

MATHEMATICS
SET - A

Time allowed : 3 hr ***Maximum Marks : 100***

General Instructions:

(i) All Question are compulsory.
*(ii) The question paper consists of 29 questions divided into three sections A, B, and C. Section A comprises of 10 questions of **one mark** each. Section B comprises of 12 questions of **four marks** each and Section C comprises of **7** questions of **six marks** each.*
(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question..
(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
*(v) Use of calculators is **not** permitted*

Section A

1 If f(x) is an invertible function, find the inverse of f(x) = $\frac{3x-2}{5}$

2 What is the value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?

3 If $A=\begin{bmatrix}5 & 4\\ 2 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & 5 & 1\\ 6 & 8 & 4\end{bmatrix}$. Write whether both AB and BA exist. If exist, then write the order.

4 Write a value of $\int\frac{1}{1+e^x}dx$.

5 Find the value of $\int_0^1 x(1-x)^2dx$

6 Let the vector $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=5$ and $|\vec{b}|=\frac{2}{5}$ and $\vec{a}\times\vec{b}$ is a unit vector. Then what is the angle between $\vec{a}$ and $\vec{b}$?

7 For what value of λ are the vectors $\vec{a}=3\hat{i}+\lambda\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+4\hat{k}$ perpendicular to each other?

8 Write the distance of the plane $\vec{r}.(2\hat{i}-\hat{j}+2\hat{k})=12$ from the origin.

9 If *A* is a square matrix of order 3 such that |adj A| = 289, find |A|.

10 If $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ satisfies $A^4=\lambda A$, then write the value of λ.

Section B

11 Show that the relation R on Z defined by
(a, b) $\in$ R $\Leftrightarrow$ a – b is divisible by 5 is an equivalence relation

12 Solve for x: $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$.

OR

If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, prove that x + y + z = xyz

13 Using the properties of determinants, prove the following: $\begin{vmatrix}y+z & x & y\\ z+x & z & x\\ x+y & y & z\end{vmatrix}=(x+y+z)(x-z)^2$

14 If $x\sqrt{1+y}+y\sqrt{1+x}=0$, find $\frac{dy}{dx}$.

OR

Discuss the continuity of the function f given by f(x) = |x +1| + |x +2| at x = -1 and x = -2

15 If $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$

16 Find the equations of normals to the curve $3x^2 - y^2 = 8$ which are parallel to the line $x + 3y = 4$.

OR

Find the intervals in which the function given by $f(x) = \sin x - \cos x$, $0 \le x \le 2\pi$ is (i) strictly increasing, (ii) strictly decreasing.

17 Evaluate the definite integral: $\int_1^4 (x^2 - x)dx$ as limit of sums.

18 Solve the following differential equation: $\frac{dy}{dx} - 2y = \cos 3x$

19 Show that the differential equation (x + y)dy + (x –y)dx = 0 is homogenous and find its particular solution, given that y = 1 when x = 1, i.e., y(1) = 1

20 Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}, \vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and is such that $\vec{d}.\vec{c} = 21$ **OR**

If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}, \vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express β in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$

21 Find the distance of the point (-1, -5, -10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$.

22 Three balls are drawn without replacement from a bag containing 5 white and 4 green balls. Find the probability distribution of the number of green balls drawn.

SECTION C

23 Using elementary transformations, find the inverse of the following matrix: $\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$

24 An Apache helicopter of enemy is flying along the curve given by $y = x^2+7$. A soldier, is placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

OR

A wire of length 36 cm is cut into two pieces. One of the pieces is turned in the form of a square and the other in the form of an equilateral triangle. Find the length of the each piece so that the sum of the areas of the two be minimum.

25 Find the area bounded by the curve $y^2 = 4a^2(x - 1)$ and the lines x = 1 and y = 4a.

26 Find the Cartesian as well as the vector equation of the planes passing through the intersection of the planes $\vec{r}.(2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r}.(3\hat{i} - \hat{j} + 4\hat{k}) = 0$, which are at unit distance from the origin.

27 A retired person has Rs. 70,000 to invest and two types of bonds are available in the market for investment. First type of bonds yields an annual income of 8% on the amount invested and the second type of bonds yields 10% per annum. As per norms, he has to invest a minimum of Rs. 10,000 in the first type and not more than Rs. 30,000 in the second type. How should he plan his investment so as to get maximum return, after one year of investment?

28 Supose that 5% of men and 0.25% of women have a grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

29 Evaluate: $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx.$ OR Evaluate: $\int_0^{3/2} |x \cos \pi x| \, dx$

Best of Luck

Answers

1 $f(y) = \frac{5y+2}{3}$ 2 π

3 AB exist 2 ×3 BA does not exist

4 $-\log(1-e^{-x}) + c$

5 $\frac{1}{12}$ 6 $\theta = \pi/6$

7 $\lambda = \frac{11}{2}$ 8 4 units

9 $|A| = 17$ 10 $\lambda = 8$

12 $x = \frac{1}{4}$

14 $\left[\frac{dy}{dx}\right] = \frac{-1}{(1+x)^2}$,Cont. at x = -1 and x = -2

15 1/a

16 x + 3y + 8 = 0 , I $(0<x<\frac{3\pi}{4})$ and

$\frac{7\pi}{4} < x < 2\pi$, D $(\frac{3\pi}{4} < x < \frac{7\pi}{4})$

17 $\frac{27}{2}$ 18 $\frac{3\sin x - 2\cos 3x}{13} + ce^{2x}$

19 $2\tan^{-1}\left(\frac{y}{x}\right) + \log\left(y^2 + x^2\right) = \frac{\pi}{2}\log 2)$

20 $\vec{d} = 7(\hat{i} - \hat{j} - \hat{k})$ 21 13

22

X	0	1	2	3
P(X)	$\frac{10}{84}$	$\frac{40}{84}$	$\frac{30}{84}$	$\frac{4}{84}$

23 $A^{-1} = \begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix}$

24 PA = $\sqrt{5}$; $\frac{144\sqrt{3}}{9+4\sqrt{3}}, \frac{324}{9+4\sqrt{3}}$

25 $\frac{16a}{3}$ 26 x-2y+2z -3 = 0

27 40,000, 30,000 28 $\frac{20}{21}$

29 $\frac{\pi}{2\sqrt{2}}\log(\sqrt{2}+1)$, $\frac{5}{2\pi} - \frac{1}{\pi^2}$