Year of Quality education 18th

Code No. **Series AG-1** CLASS XII **TMG-D/79/89**

- Please check that this question paper contains 3 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.

General Instructions: -

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 marks each, Section B is of 12 questions of 4 marks each and Section C is of 7 questions of 6 marks each.
3. Write the serial number of the question before attempting it.
4. If you wish to answer any question already answered, cancel the previous answer.
5. In questions where internal choices is provided. You must attempt only one choice.

MATHEMATICS

Time Allowed : 3 hours **Maximum Marks : 100**

PART – A

1. Show that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0.$

2. For what value of x , the matrix $\begin{bmatrix} 4 & -3 \\ x & 2 \end{bmatrix}$ is singular ?

3. Evaluate : $\int [1 + 2\tan x(\tan x + \sec x)]^{1/2} dx.$

4. Solve the differential equations : $\sqrt{1+x^2}\,dy + \sqrt{1+y^2}\,dx = 0.$

5. Find the area of the parallelogram whose adjacent sides are the vectors $\hat{i} - \hat{j} + 3\hat{k}$ and $2\hat{i} - 7\hat{j} + \hat{k}$.

6. The probability of a man hitting a target is $\frac{1}{3}$, if he fires 3 times, what is the probability of his hitting at least once ?

7. Determine the binomial distribution whose mean is 9 and whose standard deviation is 3/2.

8. Find the value of x if the area of Δ is 35 square cms with vertices (x,4),(2, -6)and (5,4).

9. The slope of the curve $2y^2 = ax^2 + b$ $at(1,-1)$ is -1. Find a and b .

10. If A and B are two events such that $P(A) = 0.3, P(B) = 0.6 \,\&\, P(B/A) = 0.5,$ find $P(A \cup B).$

PART – B

11. If $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$, Then prove that $9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$.

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12. If $y=\sin(m\sin^{-1}x)$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$.

OR

If $x^y=e^{x-y}$, prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$.

13. Evaluate $\int\frac{\sin 4x-2}{1-\cos 4x}e^{2x}dx.$

14. Find a vector whose magnitude is 3 units and which is perpendicular to the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}=3\hat{i}+\hat{j}-4\hat{k}$ and $\vec{b}=6\hat{i}+5\hat{j}-2\hat{k}$.

OR

If a unit vector $\vec{a}$ makes angles $\frac{\pi}{4}$ and $\frac{\pi}{3}$ with x –axis and y – axis respectively and an acute angle θ with z-axis, then find θ and the (scalar and vector) components of $\vec{a}$ along the axes .

15. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm/ sec . How fast is the area decreasing when the two equal sides are equal to the base?

16. The function $f(x)$ is defined as follows: $f(x)=\begin{cases} x^2+ax+b & ,0\le x\langle 2 \\ 3x+2 & ,2\le x\le 4 \\ 2ax+5b, & ,4\langle x\le 8\end{cases}$. If is continuous on $[0,8]$, find the values of a and b .

17. A letter is known to have come either from **TATANAGAR** or **CALCUTTA.** On the envelope, only the two consecutive letters TA are visible. What is the probability that the letter has come from (i) **CALCUTTA** (ii) **TATANAGAR**?

18. Evaluate: $\int_0^{\pi}\frac{x}{a^2\cos^2 x+b^2\sin^2 x}dx$.

19. Let $A=\begin{bmatrix}0 & -\tan(\alpha/2)\\ \tan(\alpha/2) & 0\end{bmatrix}$ and I be the identity matrix of order 2. Show that $I+A=(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$.

OR

Prove using property of determinant that $\begin{vmatrix}-bc & b^2+bc & c^2+bc\\ a^2+ac & -ca & c^2+ac\\ a^2+ab & b^2+ab & -ab\end{vmatrix}=(ab+bc+ca)^3$.

20. Solve the differential equation $(3xy+y^2)dx+(x^2+xy)dy=0.$

21. Using limits of sum find the integral of $\int_1^3(2x^2+5)dx.$

22. A bag contains 50 tickets numbered 1, 2, 3, ….. 50 of which five are drawn at random and arranged in ascending order of magnitude $(x_1\langle x_2\langle x_3\langle x_4\langle x_5)$. Find the probability that $x_3=30.$

OR

There are three urns A ,B and C . Urn A contains 4 white balls and 5 blue balls. Urn B contains 4 white balls and 3 blue balls. Urn C contains 2 white balls and 4 blue balls . One

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ball is drawn from each of these urns. What is the probability that out of these three balls drawn , two are white balls and one is a blue ball?

PART – C

23. Show that the matrix , $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ satisfies the equation, $A^3 - A^2 - 3A - I_3 = O.$ Hence, find A^{-1}.

24. An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

OR

Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi – vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

25. Find the area of the region $\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$.

26. Show that each of the relation R in the set A = $\{ x \in 2 : 0 \le x \le 12\}$,given by

(i)R= {(a,b): $|a-b|$. is a multiple of 4}

(ii)R= {(a,b):a = b} is an equivalence relation .Find the set of all elements to 1 in each cases.

OR

Let X be non empty set . P(X) be its power set . let * be an binary operation defined on elements of P(X) by , $A * B = A \cap B \forall A, B \in P(X)$ then (i) Prove that * is a binary operation in P(X). (ii) Is * commutative ? (iii)Is associative (iv) Find the identity element of in P(X) w.r.t. * . (v) Find all the invertible of P(x) . (vi) If $\otimes$ is another binary operation defined on P(X) as A$\otimes$B = $A \cup B$ then verify that $\otimes$ distributive over * .

27. Find the foot & image of the point (0,2,3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Also find the equation of plane contain the line and passes through (0, 2 , 3) .

28. Two godowns A and B have a grain capacity of 100 quintals and 50 quintals respectively. They supply to three shops D, E and F, whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from godowns to the shops are given below :

Transportaton cost per quintal (in Rs.)		
From / To	**A**	**B**
D	**6.00**	**4.00**
E	**3.00**	**2.00**
F	**2.50**	**3.00**

How should the supplies be transported in order that transportation cost is minimum ?

29. Define the line of shortest distance between two skew lines .Find the magnitude and the equation of the line of the shortest distance between the following lines : $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$.

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