Year of Quality education 18th

CLASS XII

- Please check that this question paper contains 3 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.

General Instructions: -

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 marks each, Section B is of 12 questions of 4 marks each and Section C is of 7 questions of 6 marks each.
3. Write the serial number of the question before attempting it.
4. If you wish to answer any question already answered, cancel the previous answer.
5. In questions where internal choices is provided. You must attempt only one choice.

MATHEMATICS

Time Allowed : 3 hours **Maximum Marks : 100**

PART – A

1. Determine whether that the relation R in the set A of human beings in a town at a particular time given by R = {(x, y) : x is wife of y} is transitive or not .
2. Write the value of $\int_0^{\pi/2} \log\left[\frac{3+5\cos x}{3+5\sin x}\right]dx$.
3. Find the distance of the point (a, b, c) from x-axis.
4. If $f(1)=4$; $f'(1)=2$, find the value of the derivative of log $f(e^x)$ w.r.t. x at the point x= 0.
5. Evaluate $\int_0^{1.5}[x]dx$ (where [x] is greatest integer function).
6. Write the order and degree of the differential equation, $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$.
7. The probability that an event happens in one trial of an experiment is 0.4. Three independent trials of the experiments are performed. Find the probability that the event happens at least once.
8. In a triangle ABC, the sides AB and BC are represented by vectors $2\hat{i}-\hat{j}+2\hat{k}$, $\hat{i}+3\hat{j}+5\hat{k}$ respectively. Find the vector representing CA.
9. Write the value of λ such that the line $\frac{x-2}{9}=\frac{y-1}{\lambda}=\frac{z+3}{-6}$ is perpendicular to the plane 3x-y-2z = 7.
10. If $A=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$, find x, $0<x<\frac{\pi}{2}$ when $A+A^T=I$.

Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph. :2337615; 4010685®, 92022217922630601(O) Mobile : 9425109601;9907757815 (P); 9300618521;9425110860(O);9993461523;9425772164

PART – B

11. Solve for x : $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x), 0<x<\frac{\pi}{2}$.

OR

Prove that : $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]=\frac{x+y}{1-xy}, |x|<1, y>0 xy<1$.

12. Evaluate: $\int x(\log x)^2 .dx$.

13. How many time must a man toss a fair coin, so that the probability of having at least one head is more than 80%?

14. If $\vec{A},\vec{B},\vec{C}$ are unit vectors, $\vec{A}\bullet\vec{B}=\vec{A}\bullet\vec{C}=0$ and angle between $\vec{B}$ and C is $\frac{\pi}{6}$, then prove that $\vec{A}=\underline{+}2(\vec{B}\times\vec{C})$.

15. Evaluate $\int_0^{\pi}\frac{xdx}{4-\cos^2 x}$.

16. Show that f(x) = $|x-3|, \forall x\in R$, is continuous but not differentiable at x=3.

OR

Differentiate the following function: $f(x)=(x\cos x)^2+(x\sin x)^{\frac{1}{x}}$.

17. Show that the curves $y^2=8x$ and $2x^2+y^2=10$ intersects orthogonally at the point (1, $2\sqrt{2}$).

OR

Verify Rolle's Theorem for the function f, given by f(x)=$e^x(\sin x-\cos x)$ on $\left[\frac{\pi}{4} \quad \frac{5\pi}{4}\right]$.

18. A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A, if 2 or 3 turns up, a ball is picked up at random from bag B, otherwise a ball is picked up from bag C. Bag A contains 3 red and 2 white balls, bag B contains 3 red and 4 white balls and bag C contains 4 red and 5 white balls. The die is rolled, a bag is picked up and a ball is drawn from it. If the ball drawn is red, what is the probability that bag B was picked up?

19. Find the particular solution of the differential equation:

$(xdy-ydx)y.\sin\left(\frac{y}{x}\right)=(ydx+xdy)x\cos\frac{y}{x}$, given that $y=\pi$ when x = 3.

20. Using properties of determinants, show that $\Delta=\begin{vmatrix}(b+c)^2 & ab & ca\\ ab & (a+c)^2 & bc\\ ac & bc & (a+b)^2\end{vmatrix}=2abc(a+b+c)^3$

21. Prove that : $\int_0^{\pi}\log(1+\cos x)dx=-\pi\log 2$.

OR

Evaluate: $\int_1^3(3x^2+e^{3x}+4)dx$, using limit of sums.

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22. A biased die is twice as likely to show an even number as an old number. The die is rolled three times. If occurrence of an even number is considered a success, then write the probability distribution of number of successes. Also find the mean number of successes.

PART – C

23. Using matrices, solve the following system of equations :

$\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4;\frac{2}{x}+\frac{1}{y}-\frac{3}{z}=0;\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2;x\neq 0,y\neq 0,z\neq 0$.

24. A right circular cone is circumscribed about a right circular, cylinder of radius 'r' and altitude 'h'. show that the volume of the cone is least, when

(i) the altitude of the cone is equal to 3 times the altitude of the cylinder.

(ii) The radius of the cone is equal to 3/2 times the radius of the cylinder.

OR

The sum of the surface areas of a rectangular parallelopiped with sides x, 2x and $\frac{x}{3}$ and a sphere given to be constant. Prove that the sum of their volume is minimum, if x is equal to three times the radius of the sphere. Find the minimum value of the sum of the volumes.

25. Find the equation of the plane containing the lines, $\vec{r}=\hat{i}+\hat{j}+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$ and $\vec{r}=\hat{i}+\hat{j}+\mu\left(-\hat{i}+\hat{j}-2\hat{k}\right)$ .Find the distance of this plane from origin and also from the point (1, 1, 1) .

OR

Find the distance of the points (3, 8, 2) from the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-2}{3}$ measured parallel to the plane $3x+2y-2z+15=0$.

26. Consider $f:\{1,2,3\}\to\{a,b,c\}$ and $g:\{a,b,c\}\to\{apple,ball,cat\}$ defined as $f(1)=a;f(2)=b,f(3)=c;g(a)=apple;g(b)=ball;g(c)=cat.$ Show that f,g and gof are invertible. Find out $f^{-1},g^{-1} and (gof)^{-1}=f^{-1}og^{-1}$.

27. A company sells two different products A and B. The two products are produced in a common reduction process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that of B is 125.Profit on each unit of A is Rs.20 and on B is Rs. 15.How many units of A and B should be produced to maximize the profit. From an L.P.P. and solve it graphically.

28. Using integration find the area bounded by the curve $|x|+|y|=1$.

29. Find the equation of the line which is parallel to the line $\frac{-x-2}{1}=\frac{y+3}{7}=\frac{2x-6}{3}$ and passing through the point (1, 2, 3) in vector form . Also find the distance between to parallel line .

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