Time: 3 Hours **MATHS TEST 2** Max. Marks: 100

General Instructions:

(1) *All questions are compulsory.*

(2) *The Question Paper consists of 29 questions divided into 3 Sections A, B and C. Section A comprises of ten questions of 1 mark each, Section B comprises of twelve questions of 4 marks each and Section C comprises of seven questions of 6 marks each*

(3) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

(4) *There is no overall choice. However, internal choice has been provided in four question of 4 marks each, two questions of 6 marks each You have to attempt only one of the alternatives in all such questions.*

(5) *Use of calculators is not permitted. However, you may ask for mathematical tables.*

SECTION – A

Q1. Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

Q2. Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a \wedge b = \min\ \{a, b\}$. Write the operation table on the operation $\wedge$.

Q3. Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \le \theta \le 2\pi$. Find range of $|A|$

Q4. In triangle ABC, the sides AB & BC are represented by vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 3\hat{j} + 5\hat{k}$ respectively. Find the vector representing CA.

Q5. Cartesian equation of a line AB are $\frac{2x-1}{2}=\frac{4-y}{7}=\frac{z+1}{2}$. Write the direction ratios of a line parallel to AB.

Q6. Construct a 3×4 matrix, whose elements are given by :

$$a_{ij}=\frac{1}{2}|-3i+j|$$

Q7. Evaluate : $\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})$

Q8. Evaluate : $\int_{-\pi/2}^{\pi/2} \sin^7 x\, dx.$

Q9. If $\vec{a}=\hat{i}+\hat{j}$; $\vec{b}=\hat{j}+\hat{k}$; $\vec{c}=\vec{k}+\hat{i}$, find a unit vector in the direction of $\vec{a}+\vec{b}+\vec{c}.$

Q10. Evaluate : $\int_{2}^{4} \frac{x}{x^2+1}dx$

<u>SECTION – B</u>

Q11. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the element of {2, 4} are related to each other. But not element of {1, 3, 5} is related to any element of {2, 4}.

OR

Let $f : N\to N$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, \text{ if n is odd} \\ \qquad\qquad \text{for all n} \in \text{ N for all n} \in \text{ N} \cdot \\ \frac{\text{n}}{2} \quad , \text{ if n is even} \end{cases}$$

State whether the function f is bijective. Justify your answer.

Q12. Show that the function defined by g(x) = x – [x] is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

Q13. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b], where a = 1 and b = 3. Find all $c \in (1,3)$ for which f'(c) = 0.

OR

For a positive constant 'a' find $\frac{dy}{dx}$, where $y = a^{t+\frac{1}{t}}$, and $x = \left(t + \frac{1}{t}\right)^a$

Q14. If a, b, c, are in A.P., find value of $\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$

Q15. Evaluate : $\int \frac{1}{\sin x(5 - 4\cos x)} dx$

OR

Evaluate : $\int_0^{\pi/2} \log(\tan x) dx.$

Q16. Evaluate : $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

Q17. Find the intervals in which the function f given by

$f(x) = \frac{4\sin x - 2x - x\cos x}{2+\cos x}$ is (i) increasing (ii) decreasing

OR

A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05cm/s. Find the rate at which its area is increasing when radius is 3.2 cm.

Q18. Show that : $\int_0^{\pi} \frac{x\sin x\,dx}{1+\sin x} = \frac{\pi}{2}(\pi - 2).$

Q19. Find whether the lines intersect

$\vec{r} = \left(\hat{i} - \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j}\right)$ *and* $\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$ or not. If intersecting, find their point of intersection.

Q20 Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right triangle. Also, find the remaining angles of the triangle.

Q21. Find x, if $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$

Q22. A manufacturer of cotter pins knows that 5% of his product is defective. If he sells cotter pins in boxes of 100 and guarantees that not more than 10 pins will be defective. What is the probability that a box will fail to meet the guaranteed quality ?

<u>SECTION – C</u>

Q23. A window is in the form of a rectangle surmounted by semicircular opening. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening.

Q24. Evaluate the following using limit of sums: $\int_1^3 (x^2 - x + e^{-x})dx.$

Q25. A person consumes two types of food A and B everyday to obtain 8 units of protein, 12 units of carbohydrates and 9 units of fat. 1 kg of food A contains 2, 6 and 1 units of protein, carbohydrate and fats respectively and food B contain 1, 1 and 3 units. Food A costs Rs.8 per kg, while food B costs Rs.5 per kg. Determine how many kgs of each food A & B should he buy to minimize the cost and still meets the minimal nutrition requirements.

Q26. In an examination, a student either guesses or copies or knows the answer to a multiple choice question with four choices each. The probability that he makes a guess is 0.35, the probability that he copies it is 1/5. The probability that answer is correct given that he copied it is 0.15. Find the probability that he (i) guesses (ii) copies the answer to the questions, given that he answered it correctly.

Q27. Find the equation of the plane determined by the points A(3, -1, 2), B(5, 2, 4) and C(-1, -1, 6). Also find the distance of the point P(6, 5, 9) from the plane.

OR

Find the coordinates of foot of perpendicular and the perpendicular distance of point P(1, 3, 4) from the plane . Also find the image of the point in the plane.

Q28. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

OR

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x - y + 2z = 1$; $2y - 3z = 1$; $3x - 2y + 4z = 2$

Q29. (i) Solve : $\sqrt{1 + x^2 + y^2 + x^2 y^2} + xy\dfrac{dy}{dx} = 0$ (ii) Form the differential equation of the family of circles having radii 3.