Sample Paper ' 2011
Class – XII
Subject – Mathematics

Time : 3 Hours Max. Marks : 100

General Instructions:

1. All questions are compulsory.

2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks .

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02questions of six marks each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION : A

1. If $a * b = 2a - b + 3$, find $2 * 3$.
2. Evaluate : $\tan^{-1}\frac{y}{x} + \tan^{-1}\frac{x-y}{x+y}$
3. If A is a square matrix of order 3 such that $|A| = 8$, find $|3A|$.
4. Evaluate : $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$
5. Evaluate : $\int_0^{\frac{\pi}{2}} \cos^2 x dx$
6. Find $\frac{dy}{dx}$ where $y = \sin^{-1}\left(4x^3 - 3x\right)$
7. Evaluate : $\int \frac{x^2 dx}{1+x^3}$
8. If $\vec{a}$ is unit vector and $\left(\vec{x}+\vec{a}\right).\left(\vec{x}-\vec{a}\right) = 8$, then find $|\vec{x}|$.
9. Find the distance of the point $(2,3,4)$ from the plane $\vec{r}.\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right) = -11$.
10. Find k if $A = \begin{bmatrix} k & 1 \\ o & 5 \end{bmatrix}$ is a singular matrix .

SECTION : B

11. Prove that the relation ' congruence modulo m ' on the set Z of all integers is an equivalence relation.

12. Prove that $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)=\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$.

OR,

Prove that $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$.

13. Without expanding at any stage , prove that $\begin{vmatrix}1 & a & a^2-bc\\ 1 & b & b^2-ca\\ 1 & c & c^2-ab\end{vmatrix}=0$.

OR,

Find the equation of the line joining $A(1,3)$ and $B(0,0)$ using determinants and find 'k' if $C(k,0)$ is a point such that area of ΔABC is 3 sq. units.

14. If for $f(x)=x^3+bx^2+ax, x\in[1,3]$, Rolle's theorem holds with $c=2+\frac{1}{\sqrt{3}}$, find a and b .

15. If $y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{bx}{(x-b)(x-c)}+\frac{c}{x-c}+1$, prove that $\frac{dy}{dx}=\frac{y}{x}\left(\frac{a}{a-x}+\frac{b}{b-x}+\frac{c}{c-x}\right)$.

OR,

If $\log\sqrt{x^2+y^2}=\tan^{-1}\frac{y}{x}$, prove that $\frac{dy}{dx}=\frac{x+y}{x-y}$.

16. Evaluate : $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$ **OR,** $\int\sqrt{\frac{1+x}{x}}dx$

17. Solve : $\frac{dy}{dx}=\cos^3 x\sin^4 x+x\sqrt{2x-1}$

OR,

In a simple circuit of resistance R , self inductance L and voltage E , the circuit i at any time t is given by $L\frac{di}{dt}+Ri=E$. If E is constant and initially no current passes through the circuit , prove that $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$

18. If $\vec{a},\vec{b},\vec{c}$ are unit vectors $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. Prove that $\vec{a}=\pm2\left(\vec{b}\times\vec{c}\right)$.

19. A die is thrown thrice . Find the mean and variance of the number of times a six is obtained .

20. Find the angle between any two diagonals of a cube .

OR,

Find the angle between the lines whose direction cosines are related by the relation $l+m+n=0$ and $l^2+m^2-n^2=0$

21. Show that $\int_0^{\pi}\frac{x\sin x}{1+\sin x}dx=\frac{\pi}{2}(\pi-2)$.

22. Let $\otimes$ be binary operation of $N\times N$ defined by $(a,b)\otimes(c,d)=(a+c,b+d)$. Show that the operation $\otimes$ is commutative as well as associative . Find the identity element for the operation $\otimes$ on $N\times N$ if any.

SECTION : C

23. Find the inverse of the matrix $\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}$ using elementary transformations.

OR,

Solve by matrix method **:** $\frac{2}{x}+\frac{3}{y}+\frac{4}{z}=-3,\frac{5}{x}+\frac{4}{y}-\frac{6}{z}=4,\frac{3}{x}-\frac{2}{y}-\frac{2}{z}=6$

24. Show that the height **'h'** of a right circular cylinder of maximum total surface area , including the two ends that can be inscribed in a sphere of radius **' r '** is given by $h^2=2r^2\left(1-\frac{1}{\sqrt{5}}\right)$.

25. Find the area of the region bounded by $\{(x,y):x^2+y^2\le 2ax,y^2\ge ax,x\ge 0,y\ge 0\}$.

OR,

Find the area bounded by the curve $y^2=4a^2(x-3)$ and the lines $x=3,y=4a$.

26. Find the length and the equation of the line of shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=z-3$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$.

27. In a test , an examinee either guesses or copies or knows the answer to a multiple choice question with four choices . The probability that he makes a guess is $\frac{1}{3}$ and that he copies the answer is $\frac{1}{6}$. When he copied the probability that it is correct is $\frac{1}{8}$. Find the probability that he knew the answer to the question , given that he correctly answered it

28. Evaluate : $\int_0^{\frac{\pi}{2}} \log(\sin x)\,dx$ **OR,** $\int \frac{\tan\theta + \tan^3\theta}{1+\tan^3\theta} d\theta$.

29. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B to go into each bottle of the drink .The chemicals are available in a prepared mix from two different suppliers . Suppliers S has a mix of 4 units of A and 2 units of B that cost Rs.10 , the suppliers T has a mix of 1 unit of A and 1 unit of B that costs Rs.4 .How many mixes from S and T should the company purchase to honour contract requirement and yet minimize cost ?

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