

CLASS – XII CBSE MATHEMATICS



TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)



CODE:- AG-2-8199

REG.NO:-TMC -D/79/89/36

General Instructions :

- All question are compulsory.
- The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
- Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
- There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one If the alternatives in all such questions.
- Use of calculator is not permitted.
- Please check that this question paper contains 65 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

सामान्य निर्देश :

- सभी प्रश्न अनिवार्य हैं।
- इस प्रश्न पत्र में 29 प्रश्न हैं, जो 3 खण्डों में अ, ब, व स है। खण्ड – अ में 10 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड – ब में 12 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको के हैं। खण्ड – स में 7 प्रश्न हैं और प्रत्येक प्रश्न 6 अंको का है।
- प्रश्न संख्या 1 से 10 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
- इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 4 प्रश्न 4 अंको में और 2 प्रश्न 6 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
- कैलकुलेटर का प्रयोग वर्जित है।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 6 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।

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Pre-Board Examination 2011 -12

Time : 3 Hours
Maximum Marks : 100
Total No. Of Pages :6

अधिकतम समय : 3
अधिकतम अंक : 100
कुल पृष्ठों की संख्या : 6

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PART – A

Q.1	Let $f : \mathbb{R} - \left\{-\frac{3}{5}\right\} \rightarrow \mathbb{R}$ be a function defined $f(x) = \frac{2x}{5x+3}$, find f^{-1} : Range of $f \rightarrow \mathbb{R} - \left\{-\frac{3}{5}\right\}$. $f^{-1}(y) = \frac{3y}{2-5y}$
Q.2	Write the value of $\int_0^{\pi/2} \log \left[\frac{3+5\cos x}{3+5\sin x} \right] dx$. $I = 0$
Q.3	For two non zero vectors \vec{a} and \vec{b} write when $ \vec{a} + \vec{b} = \vec{a} + \vec{b} $ holds. Ans a & b are like parallel vector or one of them is zero.
Q.4	A matrix A of order 3×3 has determinant 5. What s the value of $ 3A $? $3A = 135$
Q.5	Cartesian equations of a line AB are. $\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$ Write the direction ratios of a line parallel to AB. $AB = 1, -7, 2$
Q.6	A four digit number is formed using the digits 1,2,3,5 with no repetitions. Find the probability that the numbers is divisible by 5. $\frac{6}{64} = \frac{1}{4}$
Q.7	Write the order and degree of the differential equation, $y = x$ $\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$. order is 1 and degree is 2
Q.8	Evaluate , $\int_0^{1.5} [x] dx$. (where $[x]$ is greatest integer function). 0.5

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Q.9	If $4 \sin^{-1} x + \cos^{-1} x = \pi$ then find the value of x . $x = \frac{1}{2}$
Q.10	Find a, for which $f(x) = a(x + \sin x)$ is increasing. $a > 0$
PART – B	
Q.11	Evaluate : $\int \frac{2 + \sin x}{1 + \cos x} \cdot e^{x/2} \cdot dx$ $I = 2 \tan \frac{x}{2} \times e^{x/2} + c$ OR Evaluate : $\int (x+1)\sqrt{1-x-x^2} dx$. Ans $:= \frac{-1}{3}(1-x-x^2)^{3/2} + \frac{1}{8}(2x+1)\sqrt{1-x-x^2} + \frac{5}{16} \sin^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)$
Q.12	A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi – vertical angle is $\tan^{-1}(1/2)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10m. $\text{Rate of water level} = \frac{1}{5\pi} \text{ m/minute}$
Q.13	If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove that $(aI + bA)^n = a^n \cdot I + na^{n-1} bA$ where I is a unit matrix of order 2 and n is a positive integer. OR If a, b and c are real numbers and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$. Show that either $a + b + c = 0$ or $a = b = c$.
Q.14	Show that the function $y = (A + Bx)e^{3x}$ is a solution of the equation $\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 0$.

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Q.15	Find the shortest distance between the lines, whose equations are $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{10-z}{-7}$ and $\frac{x-15}{3} = \frac{58-2y}{-16} = \frac{z-5}{-5}$. Also find the angle between two lines. $\theta = \cos^{-1} \frac{-154}{\sqrt{314} \times \sqrt{98}} = -\frac{11}{\sqrt{157}}$ or $\frac{11}{\sqrt{157}}$ & S.D. = 14 OR Find the equation of the plane passing through the intersection of the planes, $2x + 3y - z + 1 = 0$; $x + y - 2z + 3 = 0$ and perpendicular the plane $3x - y - 2z - 4 = 0$. also the inclination of this plane with the xy- plane. $7x + 13y + 4z - 9 = 0, \theta = \cos^{-1} \frac{4}{\sqrt{234}}$
Q.16	Show that the differential equations $2y e^{x/y} dx + (y - 2x e^{x/y}) dy = 0$ is homogeneous and find its particular solution given that $x = 0$ when $y = 1$. Ans : $\frac{dx}{dy} = \frac{2xe^{x/y} - y}{2ye^{x/y}}$ Sol of differential equation $2e^{x/y} + \log y = c \therefore 2e^{x/y} + \log y = 2$ OR Solve the following differential equation: $(1 - x^2) \frac{dy}{dx} - xy = x^2$, given $y = 2$ when $x = 0$. $y\sqrt{1-x^2} = -\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x + 2$
Q.17	If \vec{a}, \vec{b} and \vec{c} are three unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and angle between \vec{b} and \vec{c} is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.
Q.18	Let X denote the number of colleges where you will apply after your results and $P(X = x)$ denotes your probability of getting admission in x number of colleges. It is given that $P(X = x) = \begin{cases} kx & \text{If } x = 0 \text{ or } 1 \\ 2kx & \text{If } x = 2, \\ k(5 - x) & \text{If } x = 3 \text{ or } 4 \end{cases}$ k is + ve constant .

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(a) Find the value of k.
 (b) What is the probability that you will get admission in exactly two colleges?
 (c) Find the mean and variance of the probability distribution. $(a)k = \frac{1}{8}, (b)p = \frac{1}{2}, (c) \text{mean} = \frac{19}{8}, \text{variance} = \frac{47}{64}$

Q.19 If $y = \sin^{-1}(x^2\sqrt{1-x^2} + x\sqrt{1-x^4})$ Prove that $\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}} + \frac{1}{\sqrt{1-x^2}}$.

Q.20 If $\sin^{-1} \frac{2p}{1+p^2} - \cos^{-1} \frac{1-q^2}{1+q^2} = \tan^{-1} \frac{2x}{1-x^2}$ then prove that $x = \frac{p-q}{1+pq}$.

Q.21 Evaluate : $\int_1^3 (5x^2 - e^x + 4) dx$ as a limit of sums **Ans. $\frac{154}{3} - e^3 + e$**

Q.22 Discuss the continuity and differentiability of $f(x) = \begin{cases} 1-x & x < 1 \\ (1-x)(2-x) & 1 \leq x \leq 2 \\ 3-x & x > 2 \end{cases}$. at $x = 1$ & $x = 2$. $f(x)$ is continuous at $x = 1$ and discontinuous at $x = 2$. $f(x)$ is differentiable at $x = 1$ but $f(x)$ is not differentiable at $x = 2$.

PART – C

Q.23 For $A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -6 & 9 \\ 10 & 5 & -20 \end{bmatrix}$, find A^{-1} and hence solve the system of equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$ & $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$.
 $A^{-1} = \frac{1}{1200} \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix} \therefore X = (A^{-1})^T B : x = 2, y = 3, z = 5$

Q.24 Find the probability distribution of the number of white balls drawn in a

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random draw of 3 balls without replacement from a bag containing 4 white and 6 red balls. Also find the mean and variance of the distribution.

$p(x=0) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$; $p(x=1) = 3 \times \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8}$; $p(x=2) = 3 \times \frac{4}{10} \times \frac{3}{9} \times \frac{6}{8}$; $p(x=3) = 3 \times \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8}$
 Probability distribution = $\begin{matrix} x & 0 & 1 & 2 & 3 \\ p(x) & 1/6 & 1/2 & 3/10 & 1/30 \end{matrix}$ Mean = 1.2 variance = 0.56

OR

A candidate has to reach the examination centre in time. Probability of him going by bus or scooter or by other means of transport are $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively, if he travels by bus or scooter. But he reaches in time if he uses any other mode of transport. He reached late at the centre. Find the probability that he travelled by bus.

$$\text{probability } y = \frac{\frac{3}{10} \times \frac{1}{4}}{\frac{3}{10} \times \frac{1}{4} + \frac{1}{10} \times \frac{1}{3} + \frac{3}{5} \times 0} = \frac{9}{13}$$

Q.25 Find the area of the region : $\{(x, y) : 0 \leq y \leq x^2, 0 \leq y \leq x + 2; 0 \leq x \leq 3\}$.

Ans : $\int_0^2 x^2 dx + \int_2^3 (x+2) dx = \frac{43}{6} \text{ squnit}$

OR

Find the ratio of the areas into which curve $y^2 = 6x$ divides the region bounded by $x^2 + y^2 = 16$. **ratio is $(8\pi - \sqrt{3}) : (4\pi + \sqrt{3})$**

Q.26 A point on the hypotenuse of a right triangle is at a distance 'a' and 'b' from the sides of the triangle. Show that the minimum length of the hypotenuse is $[a^{2/3} + b^{2/3}]^{3/2}$. **Ans :**

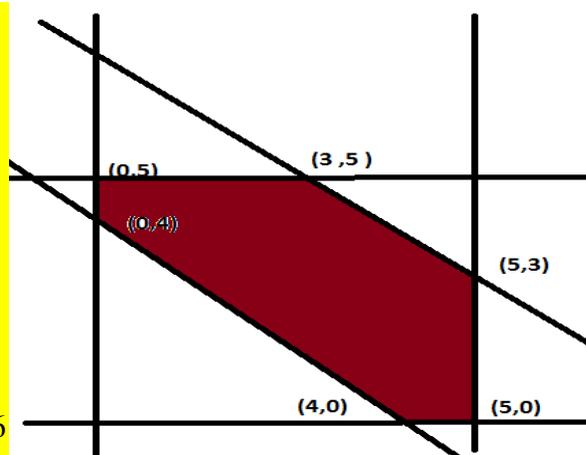
$l = a \csc \theta + b \sec \theta \therefore \tan \theta = \sqrt[3]{\frac{a}{b}} \Rightarrow \csc \theta = \frac{(a^{2/3} + b^{2/3})^{1/2}}{a^{2/3}} \& \sec \theta = \frac{(a^{2/3} + b^{2/3})^{1/2}}{b^{2/3}}$

Q.27 There is a factory located at each of the two places P & Q . From these locations , a certain commodity is delivered to each of the three depots situated at A , B & C. The weekly requirements of the depots 5 , 5 & 4

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units of commodity while the production capacity of the factories at P & Q are respectively 8 & 6 units .The cost of transportation per unit is given below. Formulate the above L.P.P. mathematically to determine how many units should be transported from each factory to each depot in order that the transportation cost is minimum.

T O	C O S T (i n ₹)		
	A	B	C
P	1 6	1 0	1 5
Q	1 0	1 2	1 0



$x \geq 0; y \geq 0; 60 - x - y \geq 0 \Rightarrow 60 \geq x + y$
 $5 - x \geq 0 \Rightarrow 5 \geq x; 5 - y \geq 0 \Rightarrow 5 \geq y; x + y \leq 6$
 $x + y - 4 \geq 0 \Rightarrow x + y \geq 4; x, y \geq 0$
 $z = x - 7y + 190$

Z at (0 , 5) = 155 ; Z at (4 , 0) = 194 ; Z at (5 , 0) = 195 ; Z at (5 , 3) = 174 ; Z at (3 , 5) = 158 ; Z at (0 , 4) = 162 ; therefore Z is minimum at (0 , 5) and minimum value = 155 there fore 0 , 5 & 3 units from the factory at p and 5 , 0 , 1 units from the factory at Q to the depots at A , B & C respectively at minmum cost of Rs 155 .

Q.28 Find the foot of the perpendicular from P(1, 2, 3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.Also obtain the equation of the plane containing the line and the point (1, 2, 3). **Foot of the perpendicular = (3,5,9) the**

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equation of plane = $18x - 22y + 5z + 11 = 0$

Q.29 Let X be a non – empty set. P(x) be its power set. Let ‘*’ be an operation defined on element of P(x) by $A * B = A \cap B \forall A, B \in P(X)$ Then,
 (i) Prove that * is a binary operation in P(X).
 (ii) Is* commutative ?
 (iii) Is* associative?
 (iv) find the identity element in P(X) w.r.t * .
 (v) find the all the invertible element of P(X)
 (vi) if O is another binary operation defined on P(X) as $A O B = A \cup B$ then verify that O distribution itself over * . **(iv) I is the identity element of p (x) $A * I = A = I * A \Rightarrow A \cap I = A = I \cap A$ (iii) Let S be invertiable element of p (x) $A * S = X = S * A \Rightarrow A \cap S = X = S \cap A \Rightarrow A = S = X(V) A O (B * C) = A \cup (B \cap C) = (A \cup B) \cap (A \cup C) = (A O B) * (A O C)$**

_____ X _____

IT'S CHOICE - NOT CHANCE - THAT DETERMINES YOUR DESTINY.