TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

20TH Year of Quality education

CODE:- AG-7-8979 **REGNO:-TMC -D/79/89/36**

Pre-Board Examination 2011 -12

Time : 3 Hours — अधिकतम समय : 3
Maximum Marks : 100 — अधिकतम अंक : 100
Total No. Of Pages :6 — कुल पृष्ठों की संख्या : 6

CLASS – XII **CBSE** **MATHEMATICS**

GENERAL INSTRUCTIONS :

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one If the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 5 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

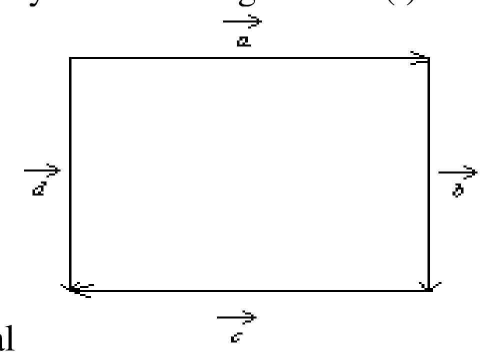
सामान्य निर्देश :

1. सभी प्रश्न अनिवार्य हैं।
2. इस प्रश्न पत्र में 29 प्रश्न है, जो 3 खण्डों में अ, ब, व स है। खण्ड – अ में 10 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड – ब में 12 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको के हैं। खण्ड – स में 7 प्रश्न हैं और प्रत्येक प्रश्न 6 अंको का है।
3. प्रश्न संख्या 1 से 10 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
4. इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 4 प्रश्न 4 अंको में और 2 प्रश्न 6 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
5. कैलकुलेटर का प्रयोग वर्जित हैं ।
6. कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ 5 हैं।
7. प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें।

PART – A

Q.1 Find the value of $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)$. Ans $=\frac{-\pi}{3}$

Q.2 In figure (a square), identify the following vectors.(i) Coinitial (ii) Equal



(iii)Collinear but not equal Ans $.(i)a\ \&\ d,(ii)b\ \&\ d,(iii)a\ \&\ c$

Q.3 Find the slope of the tangent to the curve $x=t^2+3t-8, y=2t^2-2t-5$ at the point.(2, -1) Ans $=\frac{6}{7}$

Q.4 If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k},\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ. Ans $\lambda=8$

Q.5 If f : R→ R be defined as f(x) = $\frac{3x+7}{9}$, then find $f^{-1}(x)$. Ans $f^{-1}(x)=\frac{9x-7}{3}$.

Q.6 Let relation $R=\{(x,y)\in w\times w : y=2x-4\}$. If (a , - 2) and

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	$(4,b^2)$ belong to relation R, find the value of a and b . Ans. a=1,b=2
Q.7	Find values of k if area of triangle is 4 square units and vertices are (k,0),(4,0),(0,2). Ans k=0,8
Q.8	The number of all possible matrices of order 3×3 with each entry 0 or 1 . Ans $=2^9$
Q.9	Write the total number of binary operation on a set consisting of n element . ANS n^{n^2}
Q.10	If the points (1, 1, p) and (-3, 0, 1) be equidistant from the plane $\vec{r}.(3\hat{i}+4\hat{j}-12\hat{k})+13=0$, then find the value of p. Ans $p=1,\frac{7}{3}$

PART – B

Q.11	Show that the curve $y^2=8x$ & $2x^2+y^2=10$ intersect orthogonally at the point $(1,2\sqrt{2})$. Ans $m_1\times m_2=-1$
Q.12	If $\vec{a},\vec{b},\vec{c}$ are the position vectors of the vertices A, B, C of a ΔABC respectively. Find an expression for the area of ΔABC and hence deduce the condition for the points A, B, C to be collinear. $area of \Delta ABC=\frac{1}{2}\left\|\vec{AB}\times\vec{BC}\right\| \Rightarrow A(\Delta ABC)=0 \therefore \vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}=0$
Q.13	Evaluate: $\int e^x Sin^2 4x dx$. Ans $\frac{e^x}{2}-\frac{e^x\cos 8x}{130}-\frac{4e^x\sin 8x}{65}$ OR Evaluate : $\int e^x\left(\frac{x^2+1}{(x+1)^2}\right)dx$. Ans $e^x-\frac{2e^x}{x+1}$
Q.14	Find all point of discontinuity of f , where f is defined as following : $f(x)=\begin{cases}\|x\|+3 & if x\le -3\\ -2x & -3<x<3\\ 6x+2 & if x\ge 3\end{cases}$. **Ans** $f(x)=\begin{cases}-x+3 & x\le-3\\ -2x & -3<x<3\\ 6x+2 & x\ge 3\end{cases}$ **f(x) is continous at x = - 3 Whe ; RHL=LHL = FUNCTIONAL VALUE = 6 & f(x) is not continous at x = 3 ; RHL = 20 & LHL = - 6**
Q.15	Show that the following differential equation is homogeneous, and then solve it : $ydx+x\log\left(\frac{y}{x}\right)dy-2xdy=0$. Ans $\left(1-\log\frac{y}{x}\right)=y+c$
Q.16	The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds. Ans $r=(63t+27)^{\frac{1}{3}}$ **OR** Find the particular solution of the differential equation $\frac{dy}{dx}+y\cot x=2x+x^2\cot x(x\ne0)$ given that y = 0 when $x=\frac{\pi}{2}$. Ans $y\sin x=x^2\sin x-\frac{\pi}{4}$
Q.17	Prove the following : $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]=\sqrt{\frac{1+x^2}{2+x^2}}$.
Q.18	Prove that: $\begin{vmatrix}(y+z)^2 & xy & zx\\ xy & (x+z)^2 & yz\\ xz & yz & (x+y)^2\end{vmatrix}=2xyz(x+y+z)^3$.
Q.19	The probability of India wining a test match against West Indies is 1/3. Assuming independence from match to match .Find the probability that

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in a 5 match series India's second win occurs at the third test . Ans p =1 / 3 ; q =2 / 3 Required probability;= $^{2}c_{1}\times\left(\frac{1}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{1}{3}\right)=\frac{4}{27}$

OR

A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times ,find the probability distribution of number of tails. Ans n = 3, P(H) = ¾ ,P(T) =1/ 4

x	0	1	2	4
p	27/64	27/64	9/64	1/64

Q.20 Discuss the relation R in the set of real number , defined as $R=\{(a,b): a\le b^3\}$ is Reflexive , Symmetric & Transitive . Ans ; Not reflexive $\frac{1}{2}\rangle\frac{1}{8}\Rightarrow\frac{1}{2},\frac{1}{8}\in R\therefore\left(\frac{1}{2},\frac{1}{2}\right)\notin R$; symmetric $(1,3)\in R\Rightarrow(1,3)\notin R$ & not transitive $(100,5)\in R\,\&\,(5,2)\in R\Rightarrow(100,2)\notin R$

Q.21 If $y=\frac{x\sin^{-1}x}{\sqrt{(1-x^2)}}+\log\sqrt{1-x^2}$. Prove that $\frac{dy}{dx}=\frac{\sin^{-1}x}{(1-x^2)^{3/2}}$.

OR

Prove that the derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at x = 0, is ¼ .

Q.22 Find the equation of the perpendicular drawn from the point P (2 , 4 , - 1) to the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{6-z}{9}$. Ans foot of prependicular is (-4 ,1 , -3)& Equation of perpendiculaire $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}\ or\ \frac{x+4}{6}=\frac{y-1}{3}=\frac{z+3}{2}$

PART – C

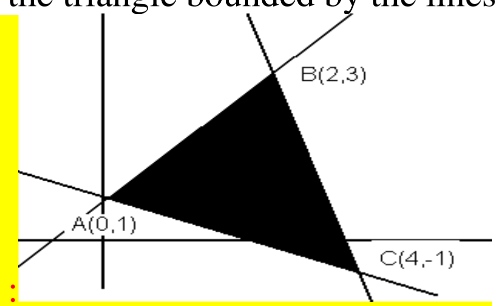
Q.23 If $A^{-1}=\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$ and B = $\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$, *find* $(AB)^{-1}$ Ans

$(AB)^{-1}=\begin{bmatrix}9 & -3 & 5\\ -2 & 1 & 0\\ 1 & 0 & 2\end{bmatrix}, B^{-1}=\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}$

Q.24 A toy manufacturers produce two types of dolls ; a basic version doll A and deluxe version doll B. Each doll of type B takes twice as long to produce as one doll of type A . The company have time to make a maximum of 2000 , dolls of type A per day , the supply of plastic is sufficient to produce 1500 dolls per day and each type requires equal amount of it .The deluxe version i.e. type B requires a fancy dress of which there are only 600 per day available . If the company makes profit of ₹ 3 and ₹ 5 per doll respectively on doll A and B , how many of each should be produced weekly in order to maximize the profit ? Solve it by graphical method. Ans : z = 3x + 5 y
$x+2y\le 2000, x+y\le 1500, y\le 600; x,y\ge 0$. corner points : (0,0) ; (1500,0) (1000, 500) (800 , 600) & (0 , 600) Thus Z is maxmium at (1000 , 500) and maximum value is 5500 .

Q.25 Evaluate: $\int_0^{\pi}\frac{x}{a^2-\cos^2 x}dx$. Ans. $\frac{\pi^2}{2a\sqrt{a^2-1}}$

Q.26 Using integration, find the area of the triangle bounded by the lines x + 2y = 2, y – x = 1 and 2x + y = 7. Ans :



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	$A_1=\int_{-1}^{3}\frac{7-y}{2}dy; A_2=\int_{1}^{3}(1+y)dy; A_3=\int_{-1}^{1}(2-2y)dy \Rightarrow A_1-A_2-A_3=6unit^2$
Q.27	A die is thrown three times. Events A and B are defined as below: A : 4 on the third throw B : 6 on the first and 5 on the second throw Find the probability of A given that B has already occurred. Ans : **Solution** The sample space has 216 outcomes. Now $A=\begin{Bmatrix}(1,1,4) & (1,2,4) & ... & (1,6,4) & (2,1,4) & (2,2,4) & ... & (2,6,4)\\ (3,1,4) & (3,2,4) & ... & (3,6,4) & (4,1,4) & (4,2,4) & ... & (4,6,4)\\ (5,1,4) & (5,2,4) & ... & (5,6,4) & (6,1,4) & (6,2,4) & ... & (6,6,4)\end{Bmatrix}$ $B=\{(6,5,1),(6,5,2),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$ and $A\cap B=\{(6,5,4)\}$. Now $P(B)=\frac{6}{216}$ and $P(A\cap B)=\frac{1}{216}$ Then $P(A\|B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{216}}{\frac{6}{216}}=\frac{1}{6}$ OR The probability of a shooter hitting a target is 3 / 4 . How many minimum number of times must he/she fire so that the probability of hitting the target at least once is more than 0.99? **Solution** Let the shooter fire n times. Obviously, n fires are n Bernoulli trials. In each trial, p = probability of hitting the target = $\frac{3}{4}$ and q = probability of not hitting the target = $\frac{1}{4}$. Then $P(X=x)={}^nC_x q^{n-x}p^x={}^nC_x\left(\frac{1}{4}\right)^{n-x}\left(\frac{3}{4}\right)^x={}^nC_x\frac{3^x}{4^n}$. Now, given that, P(hitting the target at least once) > 0.99 i.e. $P(x\geq 1)>0.99$ Therefore, $1-P(x=0)>0.99$ or $1-{}^nC_0\frac{1}{4^n}>0.99$ or ${}^nC_0\frac{1}{4^n}<0.01$ i.e. $\frac{1}{4^n}<0.01$ or $4^n>\frac{1}{0.01}=100$ The minimum value of n to satisfy the inequality (1) is 4. Thus, the shooter must fire 4 times.
Q.28	State when the line $\vec{r}=\vec{a}+\lambda\vec{b}$ is a parallel to the plane $\vec{r}.\vec{n}=d$. Show that the line $\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}+\hat{j}+4\hat{k})$ is parallel to the plane $\vec{r}.(-2\hat{i}+\hat{k})=5$. Also find the distance between the line and the plane. Ans Required Condition for line // to plane is $\vec{b}\bullet\vec{n}=0$ and distance between plane and line $\frac{7}{\sqrt{5}}$

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Q.29	Find the shortest distance of the point $(0,c)$ from the parabola $y=x^2$, where $0 \le c \le 5$. Ans $S.D. = \frac{1}{2}\sqrt{4c-1}$ OR Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone. Ans H = h – x cot α $CSA = f(x) = 2\pi RH = 2\pi x(h - x\cot\alpha)$
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	HAPPINESS IS NOTHING MORE THAN GOOD HEALTH AND A BAD MEMORY.

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