TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

CODE:- AG-10-0099 **REGNO:-TMC -D/79/89/36**

GENERAL INSTRUCTIONS :-

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 6 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

सामान्य निर्देश :

1. सभी प्रश्न अनिवार्य हैं।
2. इस प्रश्न पत्र में 29 प्रश्न है, जो 3 खण्डों में अ, ब, व स है। खण्ड – अ में 10 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड – ब में 12 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको के हैं। खण्ड – स में 7 प्रश्न हैं और प्रत्येक प्रश्न 6 अंको का है।
3. प्रश्न संख्या 1 से 10 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
4. इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 4 प्रश्न 4 अंको में और 2 प्रश्न 6 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
5. कैलकुलेटर का प्रयोग वर्जित हैं ।
6. कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ 6 हैं।
7. प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें।

Pre-Board Examination 2011 -12

Time : 3 Hours	अधिकतम समय : 3
Maximum Marks : 100	अधिकतम अंक : 100
Total No. Of Pages :6	कुल पृष्ठों की संख्या : 6

CLASS – XII **CBSE** **MATHEMATICS**

PART – A

Q.1	Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ Ans $\tan^{-1}(1)=\tan^{-1}\left[\tan\frac{\pi}{4}\right]=\frac{\pi}{4}$.
Q.2	If $\int_0^1 (3x^2+2x+k)dx=0,$ find the value of k. Ans.k = -2
Q.3	If A = $\begin{bmatrix}0 & i\\ i & 1\end{bmatrix}$ and B = $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, find the value of $\lvert A\rvert+\lvert B\rvert$. Ans $=0$
Q.4	If the binary operation * on the set of integers Z, is defined by $a*b=a+3b^2$, then find the value of 2 * 4. {Ans.50
Q.5	If $\lvert\vec{a}+\vec{b}\rvert=\lvert\vec{a}-\vec{b}\rvert$. then find the angle between $\vec{a}$ and $\vec{b}$. Ans $\frac{\pi}{2}$.
Q.6	Find the value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other {Ans. $\lambda=7$

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Q.7	Evaluate $\int \frac{dx}{x\cos^2(1+\log x)}$. Ans $I=\tan(1+\log x)+c$.
Q.8	If $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}\begin{pmatrix}3 & 1\\ 2 & 5\end{pmatrix}=\begin{pmatrix}7 & 11\\ k & 23\end{pmatrix}$, then write the value of k. {Ans.k = 17
Q.9	If A is non-singular matrix of order 3 and $\lvert adjA\rvert=\lvert A\rvert^{K}$, then write the value of k. Ans k = 2
Q.10	Find the angle between two vectors $\vec{a}\ \&\ \vec{b}$ having the same length $\sqrt{2}$ and their scalar product is -1. Ans $\vec{a}\ and\ \vec{b}=\frac{2\pi}{3}$
	PART – B
Q.11	Find the image of the point having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane $\vec{r}.(2\hat{i}-\hat{j}+\hat{k})+3=0$ Ans $-3\hat{i}+5\hat{j}+2\hat{k}$
Q.12	Evaluate $\int \frac{x^2+x+1}{(x+2)(x^2+1)}$. Ans $\frac{3}{5}\log\lvert x+2\rvert+\frac{1}{5}\left[\log\lvert x^2+1\rvert+\tan^{-1}x\right]+C$
Q.13	Show that $\frac{1}{2}\ \vec{AC}\times\vec{BD}$ represents the vector area of the plane quadrilateral ABCD. Also find the area of quadrilateral whose diagonals are $4i-j-3k\ \&-2i+j-2k$. Ans. $\frac{15}{2}unit^2$
Q.14	Is f (x) = $\lvert x-1\rvert+\lvert x-2\rvert$ continuous and differentiable at x = 1,2 . Ans : f (x) is continous at x 1 ; 2 but not differentiable at x= 1 & 2 .

Q.15	Obtain a differential equation of the family of circles touching the x-axis at origin. Ans: Equation of circle : $x^2+(y-a)^2=a^2$ Rrquired differential eqn $(x^2-y^2)y_1=2xy$
Q.16	Using properties of determinants, prove that : $\begin{vmatrix}a^2+1 & ab & ac\\ ab & b^2+1 & bc\\ ca & cb & c^2+1\end{vmatrix}=(1+a^2+b^2+c^2)$.
Q.17	Find the particular solution, satisfying the given condition, for the following differential equation . $\frac{dy}{dx}-\frac{y}{x}+\cos ec\left(\frac{y}{x}\right)=0,\ y=0 when x=1$ Ans : $\log\lvert x\rvert+\log e=\cos\frac{y}{x}\Rightarrow\log\lvert ex\rvert=\cos\frac{y}{x}$ OR Solve : $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1,\ x\neq 0$. Ans $ye^{2\sqrt{x}}=(2\sqrt{x}+c)$
Q.18	let R_+ be the set of all non-negative real numbers Let f : $R_+\to[4,\infty):f(x)=x^2+4$. Show that f is invertible that find f^{-1} Ans $f^{-1}(y)=\sqrt{y-4}$.
Q.19	Find the value of x for which $f(x)=[x(x-2)]^2$ is an increasing function. Also, find the points on the curve, where the tangents is parallel to x-axis. Ans function increase for (0,1) $\cup(2,\infty)$ &

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	Function decrease for $(-\infty,0)\cup(1,2)$ Required points are (0,0), (1,1), (2,0) OR Find the equation of the normals to the curve y = x^3+2x+6 which are parallel to the line x + 14y + 4 = 0. Ans Equation of normal at (2,18) is x + 14y + 86 =0 & Equation of normal at (-2,-6) is x + 14y -254 = 0
Q.20	A football match may be either won , drawn or lost by the host country's team . So there are three ways of forecasting the result of any match , one correct and two incorrect . Find the probability forecasting at least three correct result for four matches .**Ans: p = 1 / 3 ; q = 2 / 3 ; n = 4 ;Required probability =** $p(x=3)+p(x=4)=4\bullet\frac{2}{3}\bullet\frac{1}{27}+\frac{1}{81}=\frac{1}{9}$
Q.21	If $x=a(\cos\theta+\log\tan\frac{\theta}{2})$ & $y=a\sin\theta$, find the value of $\frac{d^2y}{dx^2}$ $at\ \theta=\frac{\pi}{4}$. Ans $\left(\frac{d^2y}{dx^2}\right)=\frac{2\sqrt{2}}{a}$ OR Differentiate w.r.t.x: $y=\frac{(2x+3)\sqrt{3x-4}}{(x^2+1)^3}$, $find\ \frac{dy}{dx}$ Ans $\frac{dy}{dx}=\frac{(2x+3)\sqrt{3x-4}}{(x^2+1)^3}\left[\frac{2}{2x+3}+\frac{3}{2(3x-4)}-\frac{6x}{(x^2+1)}\right]$

Q.22	Prove that : $2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right]=\cos^{-1}\left(\frac{b+a\cos\theta}{a+b\cos\theta}\right)$. OR Prove that : $\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)=\pi$.
	PART – C
Q.23	Find the inverse of the matrix $A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$ by using elementary row transformations. Ans $A^{-1}=\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}$
Q.24	A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, find the number of rings and chains that should be manufactured per day , so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically. {Ans z = 300 x + 190 y $x+y\le 24$; $x+\frac{1}{2}y\le 16$; $x,y\ge 0$ Z is maximum at B (8,16) i.e., x = 8, y =16. Hence 8 gold ring and 16 chains must be produced per day to get a maximum profit of Rs 5,440
Q.25	Using integration, find the area of the region bounded by the curve $x^2=4y$ and the line x = 4y -2. {Ans = $A=\int_{-1}^{2}\left(\frac{x+2}{4}-\frac{x^2}{4}\right)dx=\frac{9}{8}$ squnits OR

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Find the area bounded by the curve $y^2 = 4a^2(x-1)$ and the lines x = 1 and y = 4a. Ans $\int_0^{4a} x\,dy - \int_0^{4a} 1.dy = \frac{16a}{3}\,sq.unit.$

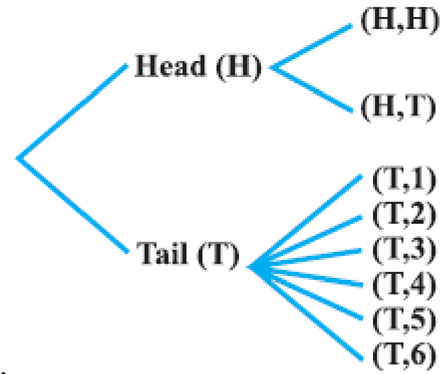
Q.26 The sum of the surface areas of a rectangular parallelepiped with side x, 2x and $\frac{x}{3}$ and a sphere gives to the constant. Prove that the sum of their volume is minimum if x is equal to three times the radius of sphere. Find the minimum value of the sum of the volumes. Ans: S = $6x^2 + 4\pi r^2$ $f(x) = \frac{2x^3}{3} + \frac{4}{3}\pi r^3$ volume is minmum at x = 3r and minmum volume is $= \frac{2}{3}\left[27r^3 + 2\pi r^3\right] = \frac{2}{3}r^3\left[27 + 2\pi\right]$

OR

A rectangle is inscribed in a semi-circle of radius ‘a’ with one of its sides on the diameter of semi-circle. Find the dimensions of the rectangle so that its area is maximum. Find the area also. Ans $f(\theta) = 2a^2\sin\theta\cos\theta = a^2\sin 2\theta$ & Area = $a^2\,sq.units$

Q.27 Evaluate : $\int_1^3 \left(2x^2 + 3x + 7\right)dx$ as limit of sums. Ans $= \frac{130}{3}$.

Q.28 Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that ‘the die shows a number greater than 4’ given that ‘there is at least one tail’.

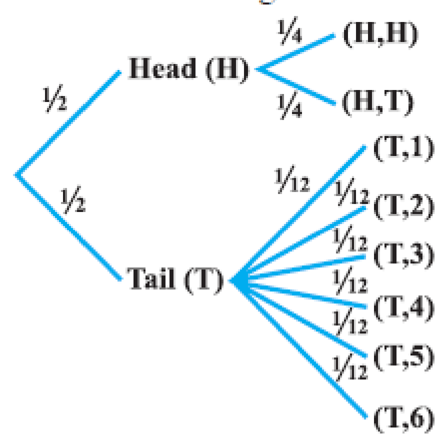


Solution The outcomes of the experiment can be represented in following diagrammatic manner called the ‘tree diagram’.The sample space of the experiment may be described as

S = {(H,H), (H,T), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)}

where (H, H) denotes that both the tosses result into head and (T, i) denote the first toss result into a tail and the number i appeared on the die for i = 1,2,3,4,5,6. Thus, the probabilities assigned to the 8 elementary events (H, H), (H, T), (T, 1), (T, 2), (T, 3) (T, 4), (T, 5), (T, 6)

are $\frac{1}{4}, \frac{1}{4}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}$ respectively which is clear from the Fig 13.2.



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Let F be the event that ‘there is at least one tail’ and E be the event ‘the die shows
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$$F = \{(H,T), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$$

$$E = \{(T,5), (T,6)\} \text{ and } E \cap F = \{(T,5), (T,6)\}$$

Now $P(F) = P(\{(H,T)\}) + P(\{(T,1)\}) + P(\{(T,2)\}) + P(\{(T,3)\}) + P(\{(T,4)\}) + P(\{(T,5)\}) + P(\{(T,6)\})$

$$= \frac{1}{4} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{4}$$

and $P(E \cap F) = P(\{(T,5)\}) + P(\{(T,6)\}) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

Hence $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{2}{9}$

Q.29 Find the equation of the plane through the intersection of planes 3x - y + 4 z = 0 and x + 3 y + 6 = 0 , whose perpendicular distance from the origin equal to 1 . Ans: Equation of plane: - x + 2y – 2z + 3 = 0 ; 2 x + y + 2 z +3 = 0

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NOTHING WILL WORK UNLESS YOU DO.

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