TARGET MATHEMATICS

THE EXCELLENCE KEY

AGYAT GUPTA (M.Sc., M.Phil.)

CODE:- AG-8-3679 **REGNO:-TMC -D/79/89/36**

GENERAL INSTRUCTIONS :

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 6 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

सामान्य निर्देश :

1. सभी प्रश्न अनिवार्य हैं।
2. इस प्रश्न पत्र में 29 प्रश्न है, जो 3 खण्डों में अ, ब, व स है। खण्ड – अ में 10 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड – ब में 12 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको के हैं। खण्ड – स में 7 प्रश्न हैं और प्रत्येक प्रश्न 6 अंको का है।
3. प्रश्न संख्या 1 से 10 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
4. इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 4 प्रश्न 4 अंको में और 2 प्रश्न 6 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
5. कैलकुलेटर का प्रयोग वर्जित हैं ।
6. कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ 6 हैं।
7. प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें।

Pre-Board Examination 2011 -12

Time : 3 Hours	अधिकतम समय : 3
Maximum Marks : 100	अधिकतम अंक : 100
Total No. Of Pages :6	कुल पृष्ठों की संख्या : 6

CLASS – XII **CBSE** **MATHEMATICS**

PART – A

Q.1	Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ-plane. Ans $\left(0,\frac{17}{2},\frac{-13}{2}\right)$
Q.2	If A is a non-singular matrix such that $A^{-1}=\begin{bmatrix}5 & 3\\ -2 & -1\end{bmatrix}$, then find $\left(A^T\right)^{-1}$, where A^T is transpose of A. Ans $(A^T)^{-1}=\begin{bmatrix}5 & -2\\ 3 & -1\end{bmatrix}$
Q.3	Write the number of all one-one functions from the set A= { a, b ,c } to itself. Ans $=6$
Q.4	In a triangle ABC, the sides AB and BC are represented by vectors $2\hat{i}-\hat{j}+2\hat{k}$, $\hat{i}+3\hat{j}+5\hat{k}$ respectively. Find the vector representing CA . Ans : -(3i+2j+7k)
Q.5	Evaluate $\int_0^1 \frac{x}{x^2+1}dx.$ Ans $I=\frac{1}{2}[\log 2-0]=\frac{1}{2}\log 2.$
Q.6	Let $A=[a_{ij}]_{m\times 3};B=[b_{ij}]_{p\times 4}\ and C=[c_{ij}]_{2\times 4}$ are such that $A_{m\times 3}.B_{p\times 4}=C_{2\times 4};$ find the value of m and p. Ans m = 2, p = 3
Q.7	Prove that : $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$.
Q.8	The vectors $\vec{a}=3\hat{i}+x\hat{j}-\hat{k}\ \&\ \vec{b}=2\hat{i}+\hat{j}+y\hat{k}$ are mutually

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	perpendicular. Given that $\|\vec{a}\| = \|\vec{b}\|$, find the values of x and y . Ans. $x = \frac{-31}{12}, y = \frac{41}{12}$
Q.9	A random variable x has the following probability distribution:

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$k+7k^2$

find the value of k. Ans /10

Q.10 Evaluate : $\int \sec^2(7-x)dx$. {Ans. $-\tan(7-x)+C$

PART – B

Q.11 Find all the point of discontinuity of the function f defined by

$$f(x) = \begin{cases} x+2 & x \le 1 \\ x-2 & 1 < x < 2 \\ 0 & x \ge 2 \end{cases}$$

. Ans :Being a polynomial function f (x) is continuous function at all point for x < 1 & 1 < x <2 To check the continuity at x = 1 & 2 . .f(x) is continous at x = 2 & discontinuous at x = 1 .

Q.12 Evaluate : $\int \frac{x^3+x}{x^4-9}dx.$ Ans : $\frac{1}{4}\log(x^4-9)+\frac{1}{12}\log\left[\frac{x^2-3}{x^2+3}\right]$

OR

Evaluate: $\int \frac{\sin x}{\sin 4x}dx$. $I = \frac{1}{4}\int \frac{dt}{(1-t^2)(1-2t^2)} = \frac{1}{4}\left[\int [\frac{-1}{1-t^2} + \frac{2}{1-t^2}]dt\right]$

$$\frac{-1}{8}\log\left|\frac{1+\sin x}{1-\sin x}\right| + \frac{1}{4\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right|$$

Q.13 Solve the differential equation : $x\frac{d^2y}{dx^2} = 1$ given that $y = 1, \frac{dy}{dx} = 0, when\ x = 1.$ Ans. $y = x\log x - x + 2$

Q.14 If $y = (\cos x)^{\log x} + (\log x)^x$; find $\frac{dy}{dx}$. Ans

$$\frac{dy}{dx} = (\cos x)^{\log x}\left[\frac{-x\tan x\log x + \log(\cos x)}{x}\right] + (\log x)^x\left[\frac{1+\log x.\log(\log x)}{\log x}\right]$$

Q.15 If a unit vector $\vec{a}$ makes angles $\frac{\pi}{4}$ and $\frac{\pi}{3}$ with x –axis and y – axis respectively and an acute angle θ with z-axis, then find θ and the (scalar and vector) components of $\vec{a}$ along the axes. Ans. $\frac{1}{\sqrt{2}}i + \frac{1}{2}j + \frac{1}{2}k \ \&\ \theta = \frac{\pi}{3}$

Q.16 Solve the equation : $\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1}b - \sec^{-1}a$ .Ans $x = \pm ab$

OR

Prove that : $\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos\left(\tan^{-1}2\sqrt{2}\right) = \frac{14}{15}$.

Q.17 Using the properties of determinants, prove the following:

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$$

Q.18 Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ?

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Solution Let E_1, E_2 and E_3 be the events that boxes I, II and III are chosen, respectively.

Then $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Also, let A be the event that 'the coin drawn is of gold'

Then $P(A|E_1) = P(\text{a gold coin from bag I}) = \frac{2}{2} = 1$

$P(A|E_2) = P(\text{a gold coin from bag II}) = 0$

$P(A|E_3) = P(\text{a gold coin from bag III}) = \frac{1}{2}$

Now, the probability that the other coin in the box is of gold

= the probability that gold coin is drawn from the box I.

$= P(E_1|A)$

By Bayes' theorem, we know that

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times 0+\frac{1}{3}\times\frac{1}{2}} = \frac{2}{3}$$

Q.19 Show that each of the relation R in the set A = $\{x \in 2 : 0 \le x \le 12\}$,given by

(i) R= {(a,b): $|a-b|$. is a multiple of 4} .Ans {1,5,9}

(ii)R= {(a,b):a = b} is an equivalence relation .Find the set of all elements to 1 in each cases. Ans {1}

Q.20 Find the intervals in which the function f given by $f(x) = \frac{4\sin x - 2x - x\cos x}{2+\cos x}$ on $[0, 2\pi]$ is (i) increasing (ii) decreasing. Ans : f (x) is increas ing on $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2},2\pi\right)$ & $\downarrow$ on $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$

OR

The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm/ sec . How fast is the area decreasing when the two equal sides are equal to the base? Ans $-\sqrt{3}bcm^2/\sec$

Q.21 Show that $y = \operatorname{cosec}^{-1}x$ is a solution of the differential equation

$$x\left(x^2-1\right)\frac{d^2y}{dx^2}+\left(2x^2-1\right)\frac{dy}{dx}=0.$$

OR

Find the general solution of the differential equation

$$: x\log x.\frac{dy}{dx}+y=\frac{2}{x}.\log x$$ {Ans $(\log x).y = 2\left[-\frac{1}{x}\log x - \frac{1}{x}\right]+c$

Q.22 By examining the chest X-ray, the probability that T.B. is detected when a person is actually suffering is 0.99.The probability that the doctor diagnoses is incorrectly that a person has T.B. on the basis of x-ray is 0.001.In a certain city , 1in 1000 persons suffers from TB. A person is selected at random and is diagnosed to have T.B. What is the chance that he actually has T.B.? Ans: Required probability =

$$\frac{\frac{1}{1000}\times .99}{\frac{1}{1000}\times .99+\frac{999}{1000}\times .001}=\frac{110}{221}$$

PART – C

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Q.23 Using integration, find the area of the triangle bounded by the lines y = 2x + 1, y = 3x +1 and x = 4. Ans Required Area $=\int_0^4 (3x+1)dx-\int_0^4 (2x+1)dx = 8unit^2$

OR

Sketch the region common to the circle $x^2+y^2=25$ and the parabola $y^2=8x$. Also , find the area of the region using integration. Ans $=\frac{2\sqrt{2}}{3}\left(\sqrt{41}-4\right)^{\frac{3}{2}}+\frac{25\pi}{2}-25\sin^{-1}\left(\frac{\sqrt{41}-4}{5}\right), sq.units.$

Q.24 Evaluate: $\int_0^{3/2} |x \cos \pi x|dx$. Ans. $\int_0^{1/2} x\cos \pi x dx-\int_{1/2}^{3/2} x\cos \pi x dx = \frac{5}{2\pi}-\frac{1}{\pi^2}$

Q.25 State the condition under which the following system of equations have a unique solutions. If A= $\begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2 \end{bmatrix}$,find A^{-1} and hence solve the following system of equations: 9x + 7y + 3z = 6; 5x – y + 4z = 1; 6x + 8y + 2z = 4 . Ans. $A^{-1}=\frac{-1}{70}\begin{bmatrix} -34 & 10 & 31 \\ 14 & 0 & -21 \\ +46 & -30 & -44 \end{bmatrix}, x=1, y=0, z=-1$

Q.26 Find the equation of the plane passing through the line of intersection of the planes x - 2y +z = 1and 2x + y + z = 8 and parallel to the line with direction ratio 1,2,1. Also find the distance of P(1,-2,-2) from this plane measured along a line parallel to r = t (i – 2j – 5k) . ans : eq of plane : 9x – 8y + 7z -21 = 0 ; point of intersection : (0 , 9 , 3) and Distance = $\sqrt{30}$

Q.27 A rectangular sheet of paper for a poster is 15000 sq. cm. in area. The margins at the top and bottom are to be 6 cm. wide and at the sides 4 cm. wide. Find the dimensions of the sheet to maximize the printed area. Ans length=138cm, breadth=92cm

OR

A square tank of capacity 250 cubic metres has to be dug out. The cost of the land is ₹ 50 per sq meter. The cost of digging increases with the depth and for the whole tank it is ₹ 400h^2, where h meters is the depth of the tank. What should be the dimension of the tank so that the cost be minimum? Ans volume of tank $=x^2h=250$; cost of land $=50x^2$; cost of digging = $400h^2$; Total cost = $50x^2+400h^2=\frac{12500}{h}+400h^2$ There fore length & breadth = 10 m , height = 2. 5m

Q.28 Find the equation of the plane parallel to line $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ and containing the lines $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ in vector and Cartesian form ,also find distance of plane from origin . Ans $x+y+z=0$, $r(i+j+k)=0$ & D = 0

Q.29 A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair in ₹ 30 while by selling one table the profit is ₹ 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically. Ans :z = 30x + 60 y $x,y\ge 0, 2x+y\le 70, x+y\le 40, x+3y\le 90$ corner points : (0,0) ; (35 , 0) ; (30 , 10) (15 , 25) ; (0 , 30) number of chair =x = 15 & table = y = 25 maximum profit = 1950

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	THE IDEAL ATTITUDE IS TO BE PHYSICALLY LOOSE AND MENTALLY TIGHT.

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