CHAPTER 10

CIRCLES

Points to Remember :

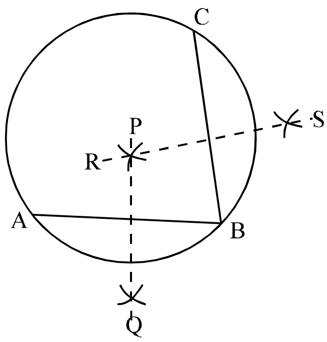
1. A **circle** is a collection of all the points in a plane, which are equidistant from a fixed point in the plane.
2. Equal chords of a circle (or of congruent circles) subtend equal angles at the centre.
3. If the angles subtended by two chords of a circle (or of congruent circles) at the centre (corresponding centre) are equal, the chords are equal.
4. The perpendicular from the centre of a circle to a chord bisects the chord.
5. The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
6. There is one and only one circle passing through three non-collinear points.
7. Equal chords of a circle (or of congruent circles) are equidistant from the centre (or corresponding centres).
8. Chords equidistant from the centre (or corresponding centres) of a circle (or of congruent circles) are equal.
9. If two arcs of a circle are congruent, then their corresponding chords are equal and conversely, if two chords of a circle are equal, then their corresponding arcs (minor, major) are congruent.
10. Congruent arcs of a circle subtend equal angles at the centre.
11. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
12. Angles in the same segment of a circle are equal.
13. Angle in a semicircle is a right angle.
14. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.
15. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.
16. If the sum of a pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

ILLUSTRATIVE EXAMPLES

Example 1. Give a method to find the centre of given circle.

Solution. Let A, B and C be any three distinct points on the given circle. Join A to B and B to C. Draw perpendicular bisectors PQ and RS of AB and BC respectively to meet at a point O.

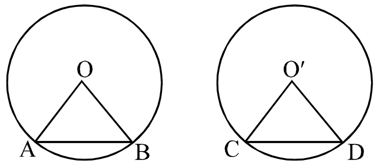
Then, O is the centre of the circle.



Example 2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal. —NCERT

Solution. Given : Two congruent circle C(O, r) and C(O′, r) such that ∠AOB = ∠CO′D.

To prove : $\overline{AB} = \overline{CD}$



Proof : In ΔAOB and ΔCO′D

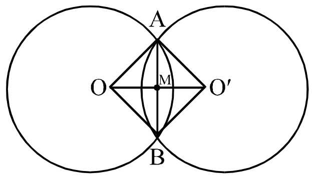
	OA = O′C	(each = r)
	OB = O′D	(each = r)
	∠AOB = ∠CO′D	(given)
⇒	ΔAOB ≅ ∠CO′D	(SAS congruence condition)
⇒	$\overline{AB} = \overline{CD}$	(cpct)

Example 3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord. —NCERT

Solution. Given : Two circles, with centres O and O′ intersect at two points A and B so that AB is the common chord of the two circles and OO′ is the line segment joining the centres of the two circles. Let OO′ intersect AB at M.

To prove : OO′ is the perpendicular bisector of AB.

Construction : Draw line segments OA, OB, O′A and O′B.



Proof : In ΔOAO′ and OBO′, we have

	OA = OB	(Radii of same circle)
	O′A = O′B	(Radii of same circle)
	OO′ = OO′	(Common side)
⇒	ΔOAO′ ≅ ΔOBO′	(SSS congruence condition)
⇒	∠AOO′ = ∠BOO′	(cpct)
⇒	∠AOM = ∠BOM	...(1)

Now, In ΔAOM and BOM, we have

	OA = OB	(Radii of same circle)
	∠AOM = ∠BOM	(from (1))
	OM = OM	(common side)
⇒	ΔAOM ≅ ΔBOM	(SAS congruence condition)
⇒	AM = BM and ∠AMO = ∠BMO	(cpct)

But, ∠AMO + ∠BMO = 180°

∴ 2∠AMO = 180° ⇒ ∠AMO = 90°

Thus, AM = BM and ∠AMO = ∠BMO = 90°

Hence, OO′ is the perpendicular bisector of AB.

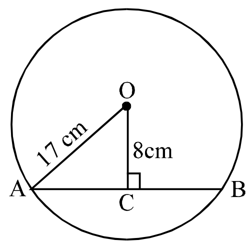
Example 4. Find the length of a chord which is at a distance of 8 cm from the centre of a circle of radius 17 cm.

Solution. Let AB be a chord of a circle with centre O and radius 17 cm.

Draw OC ⊥ AB. Join O to C.

Then, OC = 8 cm . OA = 17 cm

In right triangle OAC, using pythagoras theorem



$OA^2 = OC^2 + AC^2$

⇒ $17^2 = 8^2 + AC^2$

⇒ $AC^2 = 17^2 - 8^2$

⇒ $AC^2 = 225$

⇒ AC = 15 cm

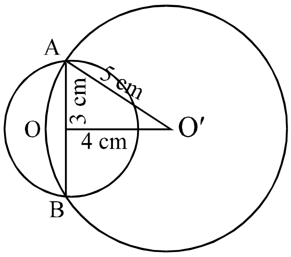
Since, perpendicular from the centre of a circle to a chord bisects a chord, we have

AB = 2 AC

= 2 × 15 cm = **30 cm Ans.**

Example 5. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord. —NCERT

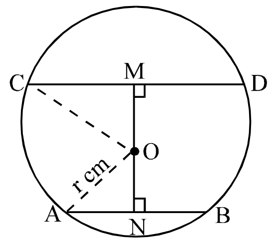
Solution. Clearly, the common chord AOB is the diameter of the cirlce with radius 3 cm.



$\therefore$ Length of common chord AOB = 2 × 3 cm **= 6 cm Ans.**

Example 6. AB and CD are two parallel chords of a circle which are on opposite sides of the centre such that AB = 10 cm, CD = 24 cm and the distance between them is 17 cm. Find the radius of the circle.

Solution. Draw $ON \perp AB$ and $OM \perp CD$.



Since, $ON \perp AB$, $OM \perp CD$ and $AB \parallel CD$

$\Rightarrow$ M, O, N are collinear points.

$\therefore$ MN = 17 cm

Let ON = x cm, then OM = $(17 - x)$ cm.

Now, we know that perpendicular from the centre of a circle to a chord bisects the chord,

$$AN = \frac{1}{2}AB = \frac{1}{2} \times 10 \text{ cm} = 5 \text{ cm, and}$$

$$CM = \frac{1}{2}CD = \frac{1}{2} \times 24 \text{ cm} = 12 \text{ cm}$$

In ΔONA, $OA^2 = ON^2 + AN^2$

$\Rightarrow \quad r^2 = x^2 + (5)^2$ ...(1)

Again, In ΔOCN, $OC^2 = OM^2 + CM^2$

$\Rightarrow \quad r^2 = (17 - x)^2 + (12)^2$ ...(2)

from (1) and (2), we get

$$x^2 + (5)^2 = (17 - x)^2 + (12)^2$$

$\Rightarrow \quad x^2 + 25 = 289 + x^2 - 34x + 144$

$\Rightarrow \quad 34x = 408 \Rightarrow x = 12$

Putting $x = 12$ in (1),

$r^2 = (12)^2 + (5)^2 = 144 + 25 = 169$

$\Rightarrow \quad r = 13$

Hence, radius of circle is **13 cm. Ans.**

Example 7. In a circle of radius 5 cm, AB and AC are two chords such that AB = AC = 6 cm. Find the length of chord BC.

Solution. Given, OA = OC = 5 cm

and AB = AC = 6 cm

Since, points A and C are equidistant from A, so AO is the perpendicular bisector of BC.

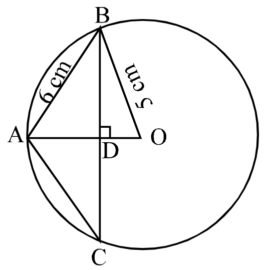
$\therefore \angle ADB = 90°$

Now, In right ΔADC,

$AC^2 = AD^2 + CD^2$

$\Rightarrow \quad (6)^2 = AD^2 + CD^2$

$\Rightarrow \quad CD^2 = 36 - AD^2 \qquad ...(1)$



Also, In ΔBDO,

$OB^2 = BD^2 + OD^2$

$\Rightarrow \quad (5)^2 = BD^2 + (AO - AD)^2$

$\Rightarrow \quad 25 = BD^2 + (5 - AD)^2$

$\Rightarrow \quad BD^2 = 25 - (5 - AD)^2$

$\Rightarrow \quad CD^2 = 25 - (5 - AD)^2 \qquad (\because \; BD = CD) \qquad ...(2)$

from (1) and (2), we get,

$36 - AD^2 = 25 - (5 - AD)^2$

$\Rightarrow \quad 36 - \cancel{AD^2} = \cancel{25} - \cancel{25} - \cancel{AD^2} + 10\,AD$

$\Rightarrow \quad AD = 3.6 \text{ cm}$

using in eq. (1),

$CD^2 = 36 - (3.6)^2$

$= 36 - 12.96$

$= 23.04$

$\therefore \quad CD = \sqrt{23.04} = 4.8 \text{ cm}$

$\therefore \quad BC = 2CD = 2 \times 4.8 \text{ cm}$

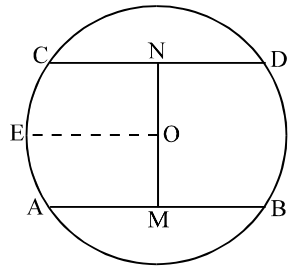
$= \mathbf{9.6 \text{ cm}}$ **Ans.**

Example 8. Prove that the line joining the mid-points of two parallel chords of a circle passes through the centre of the circle.

Solution. Given : M and N are the mid-points of two parallel chords AB and CD respectively of circle with centre O.

To prove : MON is a straight line.

Construction : Join OM, ON and draw OE || AB || CD.



Proof : Since, the line segment joining the centre of a circle to the mid point of a chord is perpendicular to the chord $\therefore$ $OM \perp AB$ and $ON \perp CD$

Now, $OM \perp AB$ and $AB \parallel OE \quad \Rightarrow OM \perp OE$

$\Rightarrow \angle EOM = 90°$

Also, $OM \perp CD$ and $CD \parallel OE \quad \Rightarrow ON \perp OE$

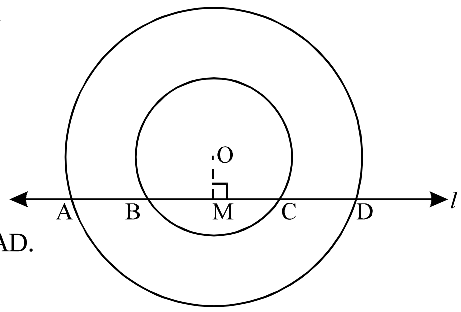
$\Rightarrow \angle EON = 90°$

$\therefore \quad \angle EOM + \angle EON = 90° + 90° = 180°$

Hence, MON is a straight line.

Example 9. In the given figure, there are two concentric circles with common centre O. *l* is a line intersecting these circles at A, B, C and D. Show that AB = CD.

Solution. Draw $OM \perp l$.



We know that perpendicular from the centre of a circle to a chord bisects a chord.

Now, BC is a chord of smaller circle and $OM \perp BC$.

$\therefore \quad BM = CM \qquad ...(1)$

Again, AD is a chord of bigger circle and $OM \perp AD$.

$\therefore \quad AM = DM \qquad ...(2)$

Subtracting (1) from (2), we get

$AM - BM = DM - CM$

$\Rightarrow \quad AB = CD$. Hence proved.

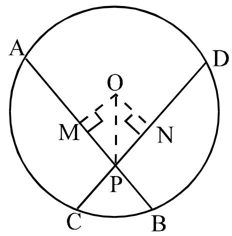
Example 10. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord. —NCERT

Solution. Given : AB and CD are chords of a circle with centre O. AB and CD intersect at P and AB = CD.

To prove : (*i*) AP = PD (*ii*) PB = CP

Construction : Draw OM $\perp$ AB and ON $\perp$ CD.

Join O to P.



Proof : $AM = MB = \frac{1}{2}AB$

($\because$ perpendicular from centre bisects the chord)

also, $CN = ND = \frac{1}{2}CD$

($\because$ perpendicular from centre bisects the chord)

But $AB = CD \Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow$ AM = ND and MB = CN ...(1)

Now, in ΔOMP and ΔONP, we have

OM = ON (equal chords of a circle are equidistant from the centre).

$\angle$OMP = $\angle$ONP (each = 90°)

OP = OP (common side)

$\Rightarrow$ $\Delta OMP \cong \Delta ONP$ (RHS congruence condition)

$\Rightarrow$ MP = PN ...(2) (cpct)

Adding (1) and (2), we get

AM + MP = ND + PN $\Rightarrow$ AP = PD

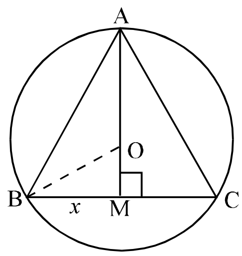
Subtracting (2) from (1), we get

MB – MP = CN – PN $\Rightarrow$ PB = CP

Hence proved.

Example 11. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone. —NCERT

Solution. Let ABC is an equilateral triangle of side 2*x* metres.



Clearly, $BM = \frac{BC}{2} = \frac{2x}{2}$ metres = *x* metres

In right ΔABM, $AM^2 = AB^2 - BM^2$

$= (2x)^2 - (x)^2 = 4x^2 - x^2 = 3x^2$

$\Rightarrow$ $AM = \sqrt{3}x$ m

Now, OM = AM – OA = $(\sqrt{3}x - 20)$ metres

In right ΔOBM, we have $OB^2 = BM^2 + OM^2$

$\Rightarrow (20)^2 = x^2 + (\sqrt{3}x - 20)^2$

$\Rightarrow 400 = x^2 + 3x^2 + 400 - 40\sqrt{3}\,x$

$\Rightarrow 4x^2 - 40\sqrt{3}\,x = 0$

$\Rightarrow 4x(x - 10\sqrt{3}) = 0$

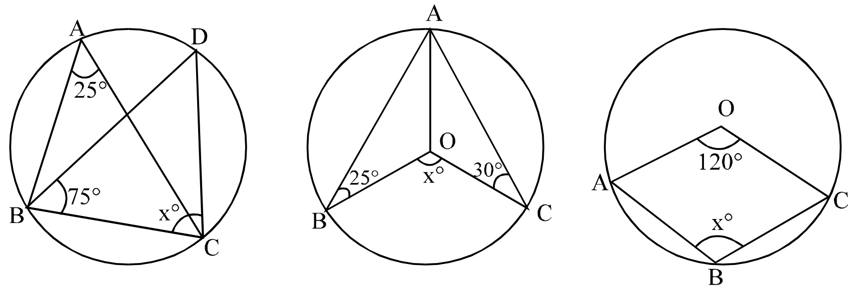
$\Rightarrow x = 0$ or $x - 10\sqrt{3} = 0$

But $x \neq 0$ $\therefore$ $x - 10\sqrt{3} = 0 \Rightarrow x = 10\sqrt{3}$ m

Now, BC = 2 MB = $2x = 2 \times 10\sqrt{3}$ m $= 20\sqrt{3}$ m

Hence, the length of each string **= 20 $\sqrt{3}$ m Ans.**

Example 12. If O is the centre of the circle, find the value of x, in each of the following figures:



(*i*) (*ii*) (*iii*)

Solution. (*i*) $\angle BAC = \angle BDC = 25°$ ($\because$ angles in same segment are equal)

Now, In ΔBCD, $\angle DBC + \angle BDC + x = 180°$ ($\because$ angles sum property of a triangle)

$\Rightarrow$ $75° + 25° + x = 180°$

$\Rightarrow$ $100° + x = 180° \Rightarrow x = 180° - 100° =$ **80° Ans.**

(*ii*) Since, OB = OA (radii of same circle)

$\therefore$ ΔOBA is an isosceles triangle

$\therefore$ $\angle OBA = \angle BAO = 25°$ ...(1)

Similarly, ΔOAC is an isosceles.

$\therefore$ $\angle OCA = \angle OAC = 30°$ ...(2)

adding (1) and (2), we get

$\angle OAB + \angle OAC = 25° + 30°$

$\Rightarrow$ $\angle BAC = 55°$

Now, $\angle BOC = 2\angle BAC$ ($\because$ The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle)

$\Rightarrow$ $x = 2 \times 55° =$ **110° Ans.**

(*iii*) Reflex $\angle AOC = 360° - 120° = 240°$

$\therefore$ $\angle ABC = \frac{1}{2}$.reflex AOC ($\because$ same as above)

$= \frac{1}{2} \times 240° = 120°$

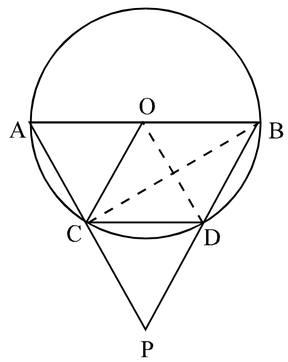
$\Rightarrow$ x = **120° Ans.**

Example 13. In the given figure, AB is a diameter of a circle with centre O and chord CD = radius OC. If AC and BD when produced meet at P, prove that $\angle APB = 60°$.

Solution. Join O to D and B to C.

Now, CD = OC = OD (radii of same circle)

$\Rightarrow$ ΔOCD is equilateral



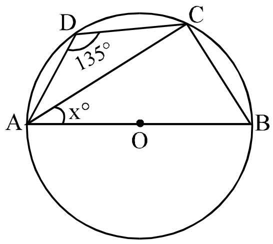
$\Rightarrow$ $\angle COD = 60°$

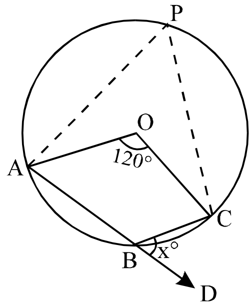
and $\angle CBD = \frac{1}{2} \times \angle COD = \frac{1}{2} \times 60° = 30°$

($\because$ angle made by $\overset{\frown}{CD}$ at centre = 2 × angle at any point on its remaining part).

Now, $\angle BCA + \angle BCP = 180°$ ($\because$ linear pair)
But, $\angle BCA = 90°$ ($\because$ angle in semi-circle)
$\Rightarrow$ $90° + \angle BCP = 180° \Rightarrow \angle BCP = 90°$
Now, in ΔBCP, $\angle BCP + \angle CBP + \angle CPB = 180°$
$\Rightarrow$ $90° + 30° + \angle CPB = 180°$
$\Rightarrow$ $\angle CPB = 180° - 120° = 60°$
$\Rightarrow$ $\angle APB = 60°$ ($\because$ $\angle CPB = \angle APB$)
Hence proved.

Example 14. In the following figures, if O is the centre of the circle, find *x*.





Solution. (*i*) $\angle ACB = 90°$ ($\because$ angle is a semi circle)
$\angle ACB = 180° - 135°$ ($\because$ opposite angles of cyclic quadrilateral are supplementary)
Now, In ΔABC, $\angle CAB + \angle ACB + \angle ABC = 180°$ ($\because$ angle sum property)
$\Rightarrow$ $x + 90° + 45° = 180°$ $\Rightarrow$ $\boldsymbol{x = 45°}$ **Ans.**
(*ii*) Take any point P on the major arc.

Now, $\angle APC = \frac{1}{2}. \angle AOC$

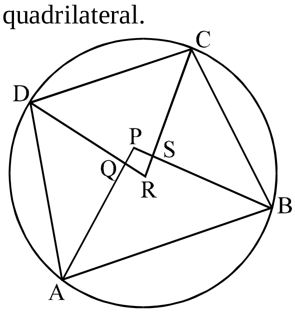
($\because$ The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle)

$= \frac{1}{2} \times 120° = 60°$.

Also, $\angle APC + \angle ABC = 180°$ ($\because$ opp. angles of cyclic quadrilateral are supplementary)
$\Rightarrow$ $60° + \angle ABC = 180°$
$\Rightarrow$ $\angle ABC = 180° - 60° = 120°$
Now, $\angle ABC + \angle DBC = 180°$ ($\because$ linear pair)
$\Rightarrow$ $120° + \angle DBC = 180°$
$\Rightarrow$ $\angle DBC = 60°$
$\Rightarrow$ $\boldsymbol{x = 60°}$ **Ans.**

Example 15. Prove that the quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.

Solution. Given : A cyclic quadrilateral ABCD in which AP, BP, CR and DR are the bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively, forming a quadrilateral PQRS.
To prove : PQRS is a cyclic quadrilateral.



Proof : In ΔAPB, $\angle APB + \angle PAB + \angle PBA = 180°$ ($\because$ angle sum property of a triangle)
Also, In ΔDRC, $\angle CRD + \angle RCD + \angle RDC = 180°$ ($\because$ same as above)

$\Rightarrow \quad \angle APB + \frac{1}{2}\angle A + \frac{1}{2}\angle B = 180° \quad$ ...(1)

and $\quad \angle CRD + \frac{1}{2}\angle C + \frac{1}{2}\angle D = 180° \quad$ ...(2)

Adding (1) and (2), we get

$$\angle APB + \angle CRD + \frac{1}{2}(\angle A + \angle B + \angle C + \angle D) = 360°$$

$\Rightarrow \quad \angle APB + \angle CRD + \frac{1}{2}(360°) = 360° \quad (\angle A + \angle B + \angle C + \angle D = 360°)$

$\Rightarrow \quad \angle APB + \angle CRD = 180°$

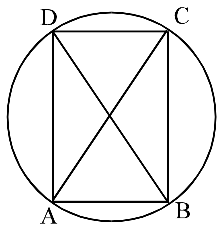
Thus, two opposite angles of quadrilateral PQRS are supplementary.

$\Rightarrow$ Quadrilateral PQRS is cyclic.

Example 16. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle. —NCERT

Solution. Given : Diagonals AC and BD of a cyclic quadrilateral are diameter of the circle through the vertices A, B, C and D of the quadrilateral ABCD.

To prove : ABCD is a rectangle.



Proof : Since AC is a diameter.

$\therefore \quad \angle ABC = 90°$ ($\because$ angle in a semi-circles is 90°)

also, quadrilateral ABCD is a cyclic.

$\therefore \quad \angle ADC = 180° - \angle ABC$

$\Rightarrow \quad \angle ADC = 180° - 90° = 90°$

Similarly, $\angle BAC = \angle BCD = 90°$.

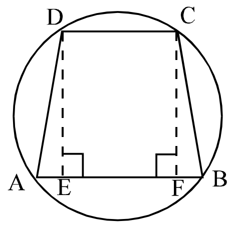
Now, each angle of a cyclic quadrilateral ABCD is 90°.

$\therefore$ ABCD is a rectangle.

Example 17. If the non-parallel sides of a trapezium are equal, prove that it is cyclic. —NCERT

Solution. Given : A trapezium ABCD in which AB || DC and AD = BC.

To prove : ABCD is a cyclic trapezium.



Construction : Draw $DE \perp AB$ and $CF \perp AB$.

Proof : In order to prove that ABCD is a cyclic trapezium, it is sufficient to prove that $\angle B + \angle D = 180°$.

Now, In ΔDEA and ΔCFB, we have

$AD = BC$ (given)

$\angle DEA = \angle CFB$ (each = 90°)

$DE = CF$ (distance between two parallel lines is always equal)

$\Rightarrow \quad \Delta DEA \cong \Delta CFB$ (RHS congruence condition)

$\Rightarrow \quad \angle A = \angle B$ and $\angle ADE = \angle BCF$ (cpct)

Now, $\quad \angle ADE = \angle BCF$

$\Rightarrow \quad 90° + \angle ADE = 90° + \angle BCF$

$\Rightarrow \angle EDC + \angle ADE = \angle FCD + \angle BCF \quad (\because \angle EDC = 90°, \angle FCD = 90°)$

$\Rightarrow \quad \angle ADC = \angle BCD$

$\Rightarrow \quad \angle D = \angle C$

Thus, $\quad \angle A = \angle B$ and $\angle C = \angle D$.

Now, $\angle A + \angle B + \angle C + \angle D = 360°$ ($\because$ sum of angles of a quadrilateral is 360°)

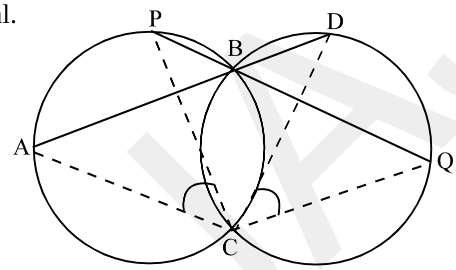
$\Rightarrow$ $2\angle B + 2\angle D = 360°$

$\Rightarrow$ $\angle B + \angle D = \dfrac{360°}{2} = 180°$

Hence, ABCD is a cyclic trapezium.

Example 18. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see figure). Prove that $\angle ACP = \angle QCD$.
—NCERT

Solution. Since angles in the same segment of a circle are equal.



$\therefore$ $\angle ACP = \angle ABP$ ...(1)

and $\angle QCD = \angle QBD$ ...(2)

But, $\angle ABP = \angle QBD$ ...(3)

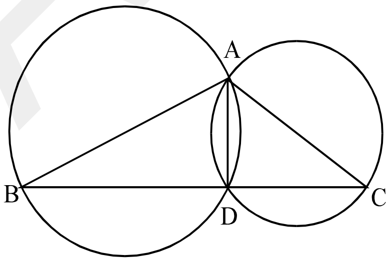
(vertically opposite angles)

from (1), (2) and (3), we get

$\angle ACP = \angle QCD$

Example 19. Two circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side. —NCERT

Solution. Given : Two circles are drawn with sides AB and AC of ΔABC as diameters. The circles intersect at D.



To prove : D lies on BC.

Construction : Join A to D.

Proof : Since AB and AC are diameters of the circles,

$\therefore$ $\angle ADB = 90°$ and $\angle ADC = 90°$

($\because$ angles in a semi-circle is 90°)

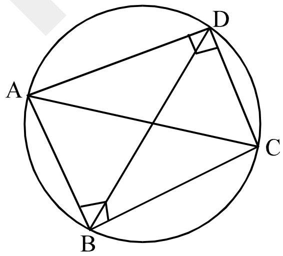
Adding, we get, $\angle ADB + \angle ADC = 90° + 90° = 180°$

$\Rightarrow$ BDC is a straight line.

Hence, D lies on BC.

Example 20. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Solution. ΔABC and ΔADC are right angled triangles with common hypotenuse AC. Draw a circle with AC as diameter passing through B and D. Join B to D. —NCERT

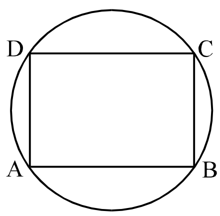


Clearly, $\angle CAD = \angle CBD$. ($\because$ angles in the same segment are equal)

Hence proved.

Example 21. Prove that a cyclic parallelogram is a rectangle. —NCERT

Solution. Given : ABCD is parallelogram inscribed in a circle.



To prove : ABCD is a rectangle.

Proof : Since ABCD is a cyclic quadrilateral,

$\therefore$ $\angle A + \angle C = 180°$ ...(1)

But, $\angle A = \angle C$ ...(2)

(opposite angles of a parallelogram are equal) from (1) and (2), we get, $\angle A + \angle A = 180°$

$\Rightarrow \quad 2\angle A = 180° \Rightarrow \angle A = 90°$

i.e. $\quad \angle A = \angle C = 90°$

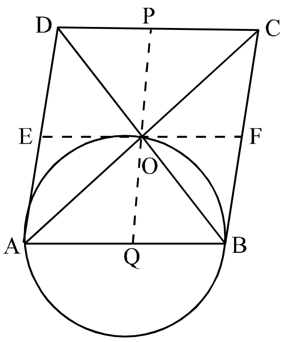
Similarly, $\quad \angle B = \angle D = 90°$

$\therefore$ ABCD is a parallelogram whose each angle is equal to 90°.

$\Rightarrow$ ABCD is a rectangle.

Example 22. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals. —NCERT

Solution. Given : ABCD is a rhombus. AC and BD are its two diagonals which bisect each other at right angles.



To prove : A circle drawn on AB as a diameter will pass through O.

Construction : From O, draw PQ || AD and EF || AB,

Proof : Since, AB = DC $\Rightarrow \frac{1}{2}$ AB = $\frac{1}{2}$ DC

$\Rightarrow$ AQ = DP

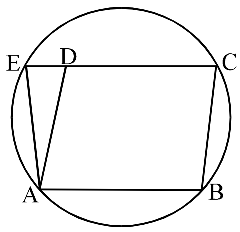
($\because$ Q and P are mid-points of AB and DC respectively)

Similarly, AE = OQ

$\Rightarrow \quad$ AQ = OQ = QB

$\Rightarrow$ A circle drawn with Q as a centre and radius AQ passes through A, O and B, which proves the desired result.

Example 23. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD. —NCERT



Solution. Since ABCE is a cyclic quadrilateral, $\angle AED + \angle ABC = 180°$ ...(1)

Now, CDE is a straight line.

$\Rightarrow \quad \angle ADE + \angle ADC = 180°$ ...(2)

($\because$ $\angle ADC$ and $\angle ABC$ are opposite angles of a parallelogram i.e. $\angle ADC = \angle ABC$)

From (1) and (2), we get

$\angle AED + \angle ABC = \angle ADE + \angle ABC$

$\Rightarrow \quad \angle AED = \angle ADE$

$\therefore$ In ΔAED, $\angle AED = \angle ADE$

$\Rightarrow$ AD = AE (sides opposite to equal angles are equal)

Hence proved.

Example 24. AC and BD are chords of a circle which bisect each other. Prove that :

(*i*) AC and BD are diameters (*ii*) ABCD is a rectangle. —NCERT

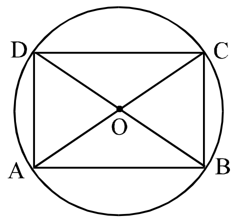
Solution. (*i*) Let AB and CD be two chords of a circle with center O.

Let they bisect each other at O.

Join AC, BD, AD and BC.

Now, In ΔAOC and ΔBOD, we have

$$OA = OB \quad (\because \text{ O is mid-point of AB})$$
$$\angle AOC = \angle BOD \quad (\text{vertically opp. angles})$$
$$OC = OD \quad (\because \text{ O is mid-point of CD})$$
$$\Rightarrow \Delta AOC \cong \Delta BOD \quad (\text{SAS congruence condition})$$
$$\Rightarrow AC = BD \quad (\text{cpct})$$
$$\Rightarrow \widehat{AO} = \widehat{BC} \quad ...(1)$$



Similarly, from ΔAOD and ΔBOC, we have

$$\widehat{AO} = \widehat{BC} \quad ...(2)$$

Adding (1) and (2), we get,

$$\widehat{AC} + \widehat{AD} = \widehat{BD} + \widehat{BC}$$
$$\Rightarrow \widehat{CAD} = \widehat{CBD}$$

⇒ CD divides the circle into two equal parts

⇒ CD is a diameter.

Similarly, AB is a diameter.

(*i*) Since, $\Delta AOC \cong \Delta BOD$ (proved above)

⇒ $\angle OAC$ i.e. $\angle BAC = OBD$ i.e. $\angle ABD$

⇒ AC || BD.

Again, $\Delta AOD \cong \Delta COB$ (proved above)

⇒ AD || CB

⇒ ABCD is a cyclic parallelogram.

⇒ $\angle DAC = \angle DBA$ ...(3) ($\because$ opp. angles of a parallelogram)

also, ABCD is a cyclic quadrilateral,

∴ $\angle DAC + \angle DBA = 180°$ ...(4)

from (3) and (4), we get

$$\angle DAC = \angle DBA = \frac{180°}{2} = 90°$$

Hence, ABCD is a rectangle.

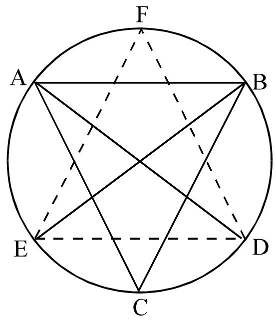
Example 25. Bisectors of angles A, B and C of a triangle ABC intersect the circumcircle at D, E and F respectively. Prove that the angles of the ΔDEF are $90° - \frac{1}{2}A$, $90° - \frac{1}{2}\angle B$ and $90° - \frac{1}{2}\angle C$. —NCERT

Solution. We have, $\angle D = \angle EDF = \angle EDA + \angle ADF$

$= \angle EBA + \angle FCA$

($\because$ $\angle EDA$ and $\angle EBA$ are in the same segment are in the same segment of a circle)

∴ $\angle EDA = \angle EBA$.



Similarly, $\angle ADF$ and $\angle FCA$ are the angles in the same segment,

∴ $\angle ADF = \angle FCA$

$$= \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{1}{2}(\angle B + \angle C)$$

$$= \frac{1}{2}(180° - \angle A) \quad [\because \angle A + \angle B + \angle C = 180°]$$

$$= 90° - \frac{1}{2}\angle A$$

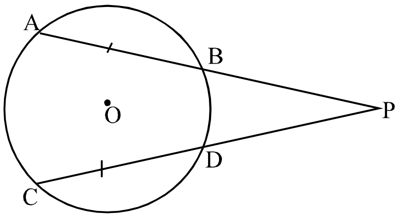
Similarly, other two angles of ΔDEF are

$$90° - \frac{1}{2}\angle B \text{ and } 90° - \frac{1}{2}\angle C.$$

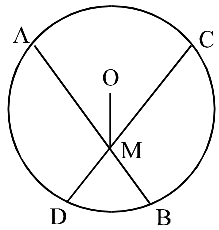
Hence proved.

PRACTICE EXERCISE

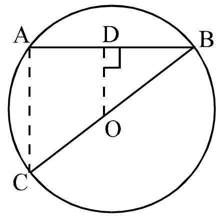
1. Show how to complete a circle if an arc of the circle is given.
2. The radius of a circle is 13 cm and the length of one of its chord is 10 cm. Find the distance of the chord from the centre.
3. AB and CD are two parallel chords of a circle which are on the opposite sides of the centre such that AB = 8 cm and CD = 6 cm. Also, radius of circle is 5 cm. Find the distance between the two chords.
4. Two chords AB and AC of a circle are equal. Prove that the centre of the circle lies on the angle bisector of $\angle BAC$.
5. If two circles intersect in two points, prove that the line through their centres is the perpendicular bisector of the common chord.
6. If a diameter of a circle bisects each of the two chords of the circle, prove that the chords are parallel.
7. In the given figure, two equal chords AB and CD of a circle with centre O, when produced meet at a point P. Prove that (*i*) BP = DP (*ii*) AP = CP.



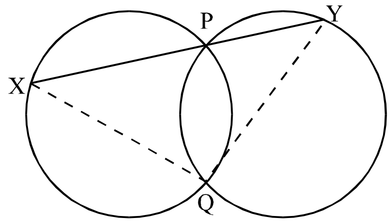
8. Two circles whose centres are O and O′ intersects at P. Through P, a line *l* parallel to OO′, intersecting the circles at C and D, is drawn. Prove that CD = 2.OO′.
9. In the given figure, O is the centre of the circle and MO bisects $\angle AMC$. Prove that AB = CD.



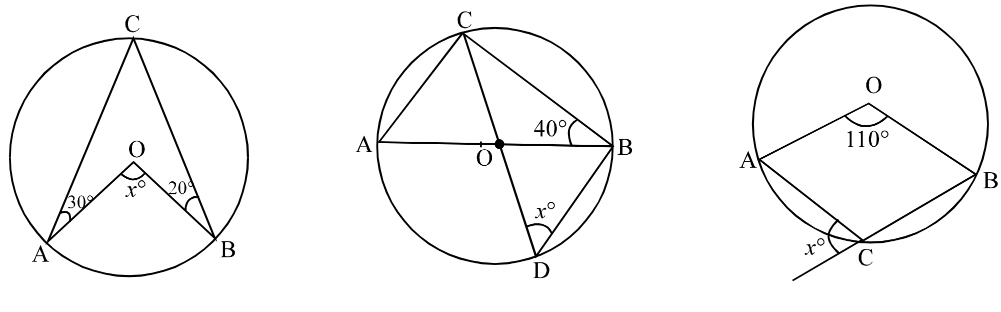
10. Show that if two chords of a circle bisect each other, they must be the diameters of the circle.
11. In the given figure, OD is perpendicular to the chord AB of a circle with centre O. If BC is a diameter, show that AC ∥ OD and AC = 2OD.



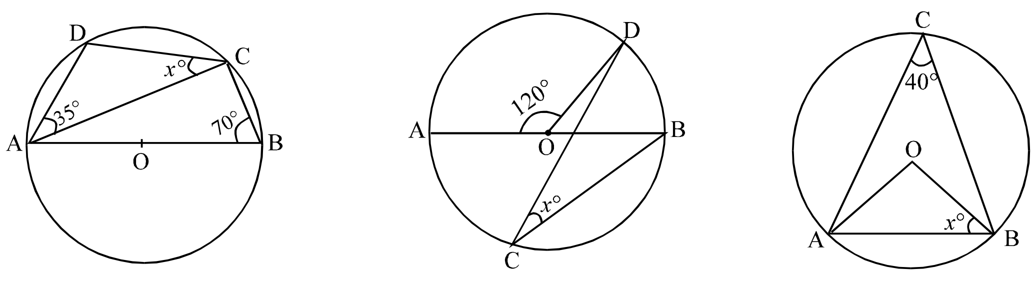
12. Prove that two different circles cannot intersect each other at more than two points.
13. Two equal circles intersect in P and Q. A straight line through P meets the circle in X and Y. Prove that QX = QY.



14. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.
15. AB and AC are two equal chords of a circle whose centre is O. If OD $\perp$ AB and OE $\perp$ AC, prove that ΔADE is an isosceles triangle.
16. Prove that angle is a semi-circle is a right angle.
17. Prove that the angles in the same segment of a circle are equal.
18. Prove that the angle formed by a chord in the major segment is acute.
19. Prove that the angle formed by a chord in the minor segment is obtuse.
20. If O is the centre of a circle, find the value of *x* in the following figures:

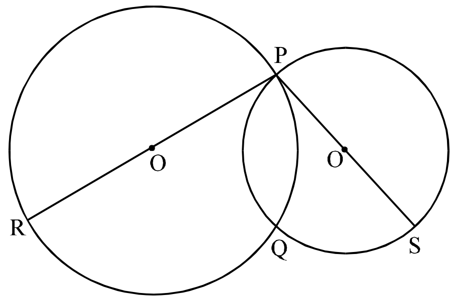


(*i*) (*ii*) (*iii*)



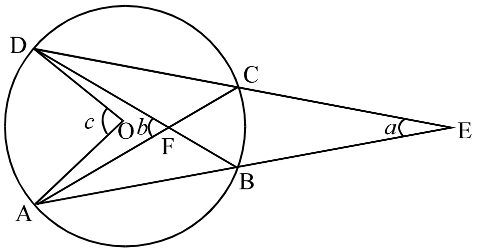
(*iv*) (*v*) (*vi*)

21. In the given figure, two circles intersect at P and Q. PR and PS are respectively the diameters of the circle. Prove that the points R, Q, S are collinear.

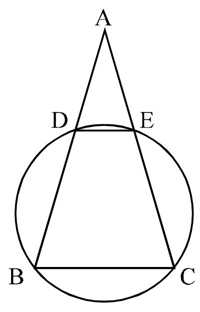


22. Prove that the circle drawn on any one of the equal sides of an isosceles triangle as diameter, bisects the third side of the triangle.

23. In the given figure, O is the centre of the circle. Prove that $\angle a + \angle b = \angle c$.

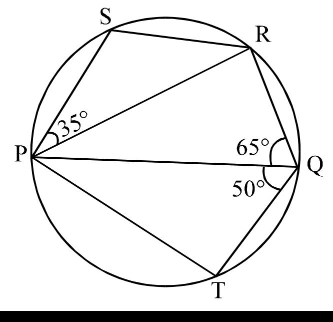


24. In an isosceles triangle ABC with AB = AC, a circle passing through B and C intersects the sides AB and AC at D and E respectively. Prove that DE || BC.

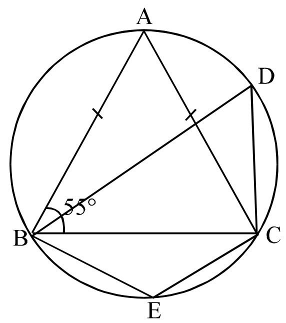


25. In the given figure, PQ is a diameter of a circle with center O. If $\angle$PQR = 65°, $\angle$SPR = 35° and $\angle$PQT = 50°, find :

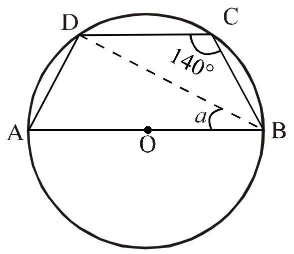
(*i*) $\angle$QPR (*ii*) $\angle$QPT (*iii*) $\angle$PRS



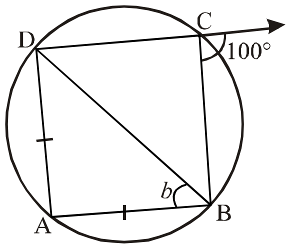
26. In the given figure ΔABC is isosceles with AB = AC and $\angle ABC = 55°$. Find $\angle BDC$ and $\angle BEC$.



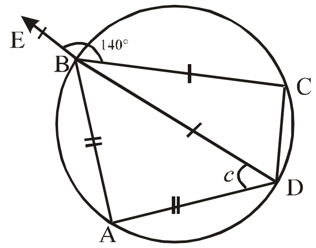
27. Find the angles marked with a letter. O is the centre of the circle.



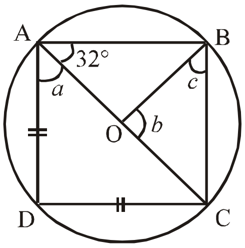
(*i*)



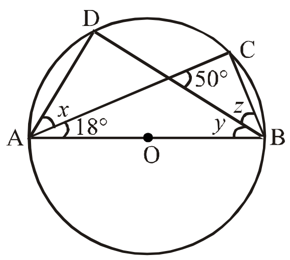
(*ii*)



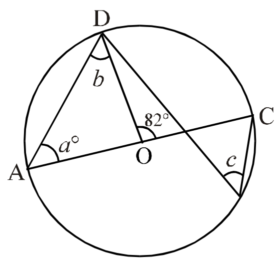
(*iii*)



(*iv*)

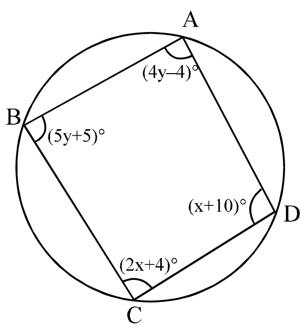


(*v*)



(*vi*)

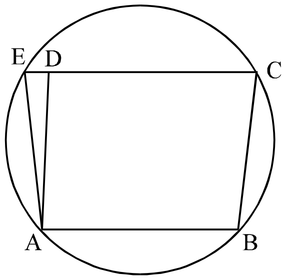
28. In the following figure, find *x* and *y*.



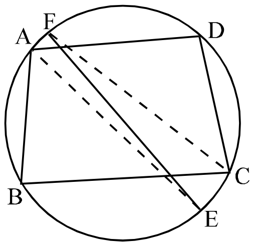
29. Prove that every cyclic parallelogram is a rectangle.
30. If two non-parallel sides of a trapezium are equal, prove that it is cyclic.
31. Prove that cyclic trapezium is always isosceles and its diagonals are equal.

32. In an isosceles ΔABC with AB = AC, a circle passing through B and C intersects the sides AB and AC at D and E respectively. Prove that DE || BC.

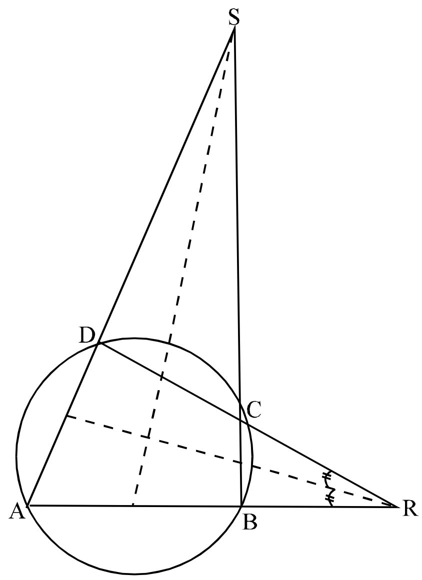
33. In the given figure, ABCD is a parallelogram. A circle through A, B, C intersects CD produced at E. Prove that AD = AE.



34. The bisectors of the opposite angles A and C of a cyclic quadrilateral ABCD intersect the circle at the points E and F respectively. Prove that EF is a diameter of the circle.



35. Prove that the angle bisectors of the angles formed by producing opposite sides of a cyclic quadrilateral (provided the are not parallel) intersect at a right angle.



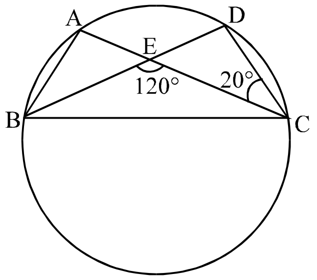
PRACTICE TEST

General Instructions :

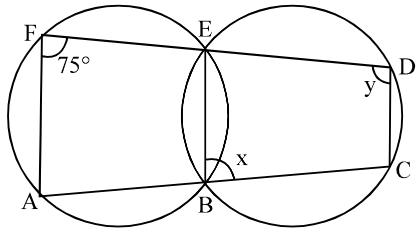
MM : 30 Time : 1 hour

Q. 1-4 carry 2 marks, Q. 5-8 carry 3 marks and Q. 9-10 carry 5 marks each.

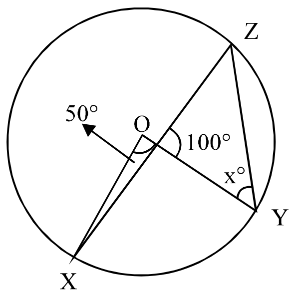
1. In the given figure, A, B, C and D are four points on the circle. AC and BD intersect at a point E such that $\angle BEC = 120°$, and $\angle ECD = 20°$. Find $\angle BAC$.



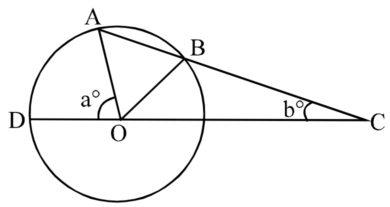
2. Prove that the line joining the mid-points of the two parallel chords of a circle passes through the centre of the circle.
3. Find the value of x and y:



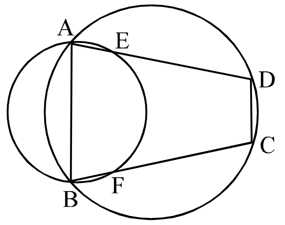
4. If O is the centre of the circle, find the value of x.



5. If two intersecting circles have a common chord of length 16 cm, and if the radii of two circles are 10 cm and 17 cm, find the distance between their centres.
6. If two non-parallel sides of a trapezium are equal, prove that it is cyclic.
7. In the given figure, AB is a chord of a circle with centre O and AB is produced to C such that BC = OB. Also, CO is joined and produced to meet the circle in D. If $\angle ACD = b°$ and $\angle AOD = a°$, prove that $a = 3b°$.

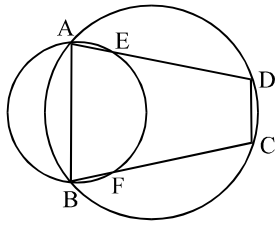


8. In the given figure, ABCD is a cyclic quadrilateral. A circle passing through A and B meets AD and BC in E and F respectively. Prove that EF || DC.



9. Prove that the quadrilateral formed by angle bisectors of a cyclic quadrilateal is also cyclic.
10. The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle. Prove it.

8. In the given figure, ABCD is a cyclic quadrilateral. A circle passing through A and B meets AD and BC in E and F respectively. Prove that EF || DC.



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10. The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle. Prove it.

ANSWERS OF PRACTICE EXERCISE

2. 12 cm

3. 7 cm

14. 6 cm

20. (*i*) 100° (*ii*) 50° (*iii*) 55° (*iv*) 35° (*v*) 30° (*vi*) 50°

25. (*i*) 15° (*ii*) 40° (*iii*) 40°

26. (*i*) 70° (*ii*) 110°

27. (*i*) $a = 50°$ (*ii*) $b = 40°$ (*iii*) c = 35°
(*iv*) $a = 45°, b = 64°, c = 58°$ (*v*) $x = 40°, y = 32°, z = 40°$ (*vi*) $a = 41°, b = 41°, c = 41°$

28. $x = 40°, y = 25°$

ANSWERS OF PRACTICE TEST

1. 100° **3.** $x = 75°, y = 105°$ **4.** 55° **5.** 21 cm