[Model Test Paper /Cl–XII(CBSE'13)/27th Jul'12] MODEL TEST [FM–100/Time–3 hrs.]

INSTRUCTIONS:

i) All questions are compulsory.

ii) The question paper consists of **29** questions divided into Three sections A, B and C. Section A comprises of **10** questions of **one mark** each, Section B comprises of **12** questions of **four marks** each, and Section C comprises of **7** questions of **six** marks each.

iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirements of the question.

iv) There is no overall choice. However, internal choice has been provided in **4 questions of four marks each** and **2 questions of six marks each**. You have to attempt only one of the alternatives in all such questions.

iv) Use of calculators is not permitted.

SECTIONS – A

1. If * be a binary operation defined by a*b = 2a + b – 3, then find 3*4. [1]

2. Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$. [1]

3. A square matrix A, of order 3, has |A|=5, find |A.adjA|. [1]

4. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$ [1]

5. For what value of x, is the matrix $\begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular ? [1]

6. Prove that he function $-\frac{x^3}{3}+x^2-x+\frac{3}{2}$ is decreasing in R. [1]

7. Evaluate: $\int\frac{3\cos x}{2\sin^2 x}dx$ [1]

8. If $\vec{a}=\hat{i}+2\hat{j}-\hat{k}$ *and* $\vec{b}=3\hat{i}+\hat{j}-5\hat{k}$, then find a unit vector in the direction of $\vec{a}-\vec{b}$. [1]

9. Find a vector of magnitude $\frac{5}{2}$ units which is parallel to the vector $3\hat{i}+4\hat{j}$. [1]

10. If P(1, 5, 4) and Q (4, 1, –2), then find the direction ratio of $\overrightarrow{PQ}$. [1]

SECTIONS – B

11. Let f, g : R $\to$R be defined as f(x) = |x| and g(x) = [x], where [x] denotes greatest integer less than or equal to x. Then evaluate $\frac{(gof)\left(-\frac{5}{3}\right)-(fog)\left(-\frac{5}{3}\right)}{(fo(gof))\left(-\frac{5}{3}\right)}$. [4]

12. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, $-\frac{\pi}{2}<x<\frac{3\pi}{2}$ in the simplest form. [4]

OR, Solve : $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$, $x>0$.

13. Prove that, $\begin{vmatrix} 1+a^2 & ab & ac \\ ab & 1+b^2 & bc \\ ca & cb & 1+c^2 \end{vmatrix} = (1+a^2+b^2+c^2)$. [4]

OR, Prove that, $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$

14. Find the values of 'a' and 'b' such that the function defined by $f(x) = \begin{cases} 5 & if\ x \le 2 \\ ax+b & if\ 2 < x < 10 \\ 21 & if\ x \ge 10 \end{cases}$

is a continuous function. [4]

15. If $y^2 = 4ax$, prove that, $\frac{d^2y}{dx^2} \cdot \frac{d^2x}{dy^2} = -\frac{2a}{y^3}$ [4]

OR, Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line $y - 4x + 5 = 0$.

16. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find te approximateerror in calculating its volume. [4]

OR, Find the intervals in which the function f given by $f(x) = \sin x + \cos x, \quad 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

17. Evaluate $\int \frac{dx}{\sin(x-\alpha)\sin(x-\beta)}$ [4]

18. Evaluate $\int_0^{\pi} \frac{x.\sin x}{1+\cos^2 x}.dx$ [4]

19. Evaluate $\int \frac{(x+x^3)^{\frac{1}{3}}dx}{x^4}$. [4]

20. Let $\hat{a} = 2\hat{i} + \hat{k}$, $\hat{b} = \hat{i} + \hat{j} + \hat{k}$ and $\hat{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ be three vectors. Find a vector $\hat{r}$ which satisfies $\hat{r} \times \hat{b} = \hat{c} \times \hat{b}$ and $\hat{r} \cdot \hat{a} = 0$. [4]

21. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x + 3y + 4z - 12 = 0$. [4]

22. Find the probability distribution of number of heads in two tosses of a coin. [4]

SECTIONS – C

23. Using elementary operations, find the inverse of $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$, if it exists. [6]

24. A point P is given on the circumference of a circle of radius r. The chord QR is parallel to the tangent line at P. Find the maximum area of the triangle ΔPQR. [6]

25. Using integration, find the area of the circle $x^2 + y^2 = 16$, which is exterior to the parabola $y^2 = 6x$. [6]

OR, Using integration, find the area of the region bounded by the triangle whose vertices are (1, 3), (2,5) and (3, 4)

26. Solve the differential equation : $(\tan^{-1}y - x)\, dy = (1+y^2)dx$ [6]

27. Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes $\vec{r}.\left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5$ and $\vec{r}.\left(3\hat{i} + \hat{j} + \hat{k}\right) = 6$. [6]

OR, Find the perpendicular distance of the point (2, 3, 4) from the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also find the coordinates of the foot of the perpendicular.

28. A company sells two different products A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that of B is 125. Profit of each unit of A is `20 and on B is `15. How many units of A and B should be produced to maximize the profit ? Form an L.P.P and solve it graphically. [6]

29. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that was produced by A ? [6]

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"Learning is a Treasure,
which accompanies its owner everywhere."