REVISION TEST SERIES - 01

CLASS XII [2012 – 2013]

Max.Marks: 100 **Time Allowed: 180 Minutes**

SECTION – A

(Question numbers 01 to 10 carry one mark each.)

Q01. Find the equation of tangent to the curve $y = x + \frac{4}{x^2}$ which is parallel to X- axis.

Q02. Prove that: $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = 3\tan^{-1}\frac{x}{a}$.

Q03. Write the value of $\int \frac{\cos x \sin x}{a^2 \sin^2 x + b^2 \cos^2 x} dx$.

Q04. If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, write the values of x and y.

Q05. What is tangent of the angle which the vector $\hat{i} + \hat{j} + \sqrt{2}\,\hat{k}$ makes with the Z-axis?

Q06. Elements of a matrix A of order 2×2 are given by $a_{ij} = \frac{(i+2j)^2}{2}$. Write the value of element a_{21}.

Q07. Prove that: $\int_0^p f(x)dx = \int_0^p f(p-x)dx$.

Q08. Solve for x: $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$.

Q09. Find the area of a parallelogram having diagonals $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} - 4\hat{k}$.

Q10. Discuss the divisibility of Δ such that $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$; $x \neq 0,\ y \neq 0$.

SECTION – B

(Question numbers 11 to 22 carry four marks each.)

Q11. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then find a vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a}.\vec{b} = 3$.

OR If a vector $\vec{a}$ makes an angle of $\pi/4$ with the positive directions of each of X and Y- axis, then find the angle which is made by it with positive direction of Z- axis. Hence write the unit vector of $\vec{a}$.

Q12. Solve the differential equation: $e^x \tan y\, dx + \sec^2 y\,(1 - e^x)\, dy = 0$.

Q13. A car starts from a point P at time $t = 0$ seconds and stops at point Q. The distance x, in the metres, covered by it, in t seconds is given by $x = t^2\left(2 - \frac{t}{3}\right)$. Find the time taken by it to reach Q and also find distance between the points P and Q.

OR Find the intervals in which $\tan^{-1}(\sin x + \cos x)$ is rising and/or falling.

Q14. Suppose 15% of men and 36% of women have grey hair. The probability of dying hair by men is 21% and by women is 63%. A dyed hair person is selected at random, what is the probability that this person is a women? Excessive use of dyes to colour the hair can prove harmful. Elaborate.

Q15. Form the differential equation of family all circles passing through origin and having their centers on X- axis.

Q16. Find the value of 'c' for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3].

OR If $f(x) = \frac{\log x}{x}$ then, show that $f''(x) = \frac{2\log x - 3}{x^3}$.

Q17. Find the angle between the pair of lines: $\frac{2-x}{-2}=\frac{y-1}{7}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$. Also check whether the lines are parallel or perpendicular.

Q18. Find x such that: $\int_{\sqrt{2}}^{x}\frac{dt}{t\sqrt{t^2-1}}=\frac{\pi}{2}$. **OR** Evaluate: $\int\frac{dx}{\cos x+\sqrt{3}\sin x}$.

Q19. Using properties of determinants, prove that: $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}=\left(a+b+c\right)^3$.

Q20. Prove that $\sin y=\tan^2\left(\frac{x}{2}\right)$, if it is given that $y=\cot^{-1}\left[\sqrt{\cos x}\right]-\tan^{-1}\left[\sqrt{\cos x}\right]$.

Q21. Let the function $f:\mathrm{N}\to\mathrm{N}$ is defined as: $f(n)=\begin{cases}\frac{n+1}{2}, \text{ when } n \text{ is odd}\\ \frac{n}{2}, \text{ when } n \text{ is even}\end{cases}$ for all $n\in\mathrm{N}$.

State whether the function f is bijective function. Justify your answer.

Q22. If $f(x)=\begin{cases}x^2+\mathrm{a}x+\mathrm{b}, \text{ if } 0\le x\le 2\\ 3x+2, \text{ if } 2\le x\le 4\\ 2\mathrm{a}x+5\mathrm{b}, \text{ if } 4\le x\le 8\end{cases}$ is a continuous function on the interval [0,8] then, find the value(s) of 'a' and 'b'.

SECTION – C

(Question numbers 23 to 29 carry six marks each.)

Q23. Find the area bounded between the curves $y^2=x$ and $y=|x|$.

Q24. A dealer wishes to purchase a number of fans and sewing machines. He has only ₹5760 to invest and has space for at most 20 items. A fan costs him ₹340 and a sewing machine costs him ₹260. His expectation is that he can sell a fan at a profit of ₹22 and sewing machine at ₹18. Assuming that he can sell all the items he buys, how he should invest his money in order to maximize his profit?

Q25. A window is in the form of a rectangle surmounted by a semicircular opening. Total perimeter of the window is 10m. Find the dimensions of window to admit maximum light through it.

OR A 20m steel wire is to be cut into two pieces. The first piece is transformed into a circle and the other one into an equilateral triangle. Find out what should be the length of two pieces so that the combined area of both is minimum?

Q26. Three shopkeepers A, B and C go to a store to buy electric equipments. A purchases 12 dozens 100watt bulbs, 5 dozen tube-lights and 6 dozens CFLs. B purchases 10 dozens 100watt bulbs, 6 dozen tube-lights and 7 dozens CFLs. Also, C purchases 11 dozens 100watt bulbs, 13 dozen tube-lights and 8 dozens CFLs. One 100watt bulb costs ₹40, one tube-light costs ₹65 and one CFL costs ₹120. Use matrix multiplication to calculate each individual's bill.

In your opinion, which equipment would you prefer to buy and why? Give two reasons.

Q27. Find the equation of the plane which contains the line of intersection of the planes given as: $\vec{r}.(\hat{\mathrm{i}}+2\hat{\mathrm{j}}+3\hat{\mathrm{k}})=4$ and $\vec{r}.(2\hat{\mathrm{i}}+\hat{\mathrm{j}}-\hat{\mathrm{k}})+5=0$ and which is perpendicular to the plane given as $\vec{r}.(5\hat{\mathrm{i}}+3\hat{\mathrm{j}}-6\hat{\mathrm{k}})+8=0$.

Q28. A bag contains five bananas, four oranges and three guavas. Three fruits are taken from it one after the other. Find the probability distribution of number of oranges if the fruits were taken

(i) with replacement and, **(ii)** without replacement.

Intake of fruits is beneficial for health. Comment in short.

Q29. Evaluate $\int_0^2(3^x+5x^2+7x-2)\,dx$ as limit of sums. **OR** Evaluate: $\int_{-1/2}^{1/2}\left|x\cos\frac{\pi x}{2}\right|dx$.