

**Sample Paper - 2013**  
**Sub: MATHEMATICS**  
**Class: XII**

**General Instructions :**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

**SECTION – A**

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**Question numbers 1 to 10 carry 1 mark each.**

1. Write the value of  $\sin(\cot^{-1} x)$
2. Evaluate :  $\sin^{-1} \frac{1}{3} - \cos^{-1} \left( -\frac{1}{3} \right)$
3. If  $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ , find  $AA'$ .
4. If  $A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$ , find  $A^{16}$ .
5. If A be a non singular square matrix of order 3 and  $|A| = 5$ , then find  $|\text{adj}A|$ .
6. Find the value of  $\int_2^8 |x - 5| dx$
7. Evaluate :  $\int \frac{x^2 dx}{1+x^3}$
8. Find the projection of the vector  $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$  on the vector  $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$
9. Find the angle between the vectors  $\vec{a} = \hat{i} - \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ .
10. If the value of x if  $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$ .

## SECTION – B

**Question numbers 11 to 22 carry 4 marks each.**

11. Let  $N$  denote the set of all natural numbers and let  $R$  be a relation on  $N \times N$ , defined by  $(a,b)R(c,d) \Leftrightarrow ad = bc$  for all  $(a,b),(c,d) \in N \times N$ . Show that  $R$  is an equivalence relation .

12. Write the simplest form of  $\sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$ .

13 . Express the matrix  $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$  as the sum of two matrices such that one is symmetric and other is skew symmetric.

**OR,**

Using properties of determinant , prove that  $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$

14. If  $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ , prove that  $\frac{dy}{dx} = \frac{x-1}{2x(x+1)}$

15. Sand is pouring from a pipe at the rate of  $12 \text{ cm}^3/\text{s}$ . The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is  $4 \text{ cm}$ ? Write two uses of sand in our life ?

16. If the function  $f(x)$  defined by

$$f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

Is continuous at  $x = 0$

17. Evaluate :  $\int \frac{\log x}{(1+\log x)^2} dx$  **OR**  $\int x \tan^{-1} x dx$

18. Evaluate :  $\int \frac{\tan x}{a+b \tan^2 x} dx$  **OR**  $\int \frac{x^2-1}{x^4+1} dx$

19. Evaluate using properties of definite integral :  $\int_0^1 \cot^{-1}(1-x+x^2) dx$

20. The scalar product of the vector  $\hat{i} + \hat{j} + \hat{k}$  with the unit vector along the sum of vectors  $2\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$  is equal to one . Find the value of  $\lambda$  .

21. Find the equation of the plane through the points (2,-3,1) and (5,2,-1) and perpendicular to the plane  $x - 4y + 5z + 2 = 0$ .

22. A die is thrown 6 times . If getting an odd number is success , what is the probability of (i) 5 successes (ii) at most 5 successes . What is the shape of a die ?

### SECTION – C

Question numbers 23 to 29 carry 6 marks each.

23. If  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$  are two square matrices , find AB and hence solve the system of linear equations :  $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$ .

OR,

Using elementary row transformation , find the inverse of the matrix  $\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$ .

24. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

25 Find the area of the smaller region bounded by  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and  $\frac{x}{a} + \frac{y}{b} = 1$

26 Show that the differential equation  $2ye^{\frac{x}{y}}dx + \left( y - 2xe^{\frac{x}{y}} \right) dy = 0$  homogeneous and find its particular solution, given that ,  $x = 0$  when  $y = 1$ .

27 .Show that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x-4}{5} = \frac{y+1}{5} = z$  intersect . Also find the point of intersection .

28. A cooperative society of farmers has 50 hectares of land to grow two crops X and Y . The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximise the total profit of the society? Write the advantage of cooperative society in a village ?

29. Suppose that the reliability of a HIV test is specified as follows :  
 Of people having HIV , 90% of the test detect the diseases but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV-ive but 1% are diagnosed as showing HIV+ive. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ive. What is the probability that the person actually has HIV? What is the full meaning of HIV?

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