PLEASURE TEST REVISION SERIES XII – 10

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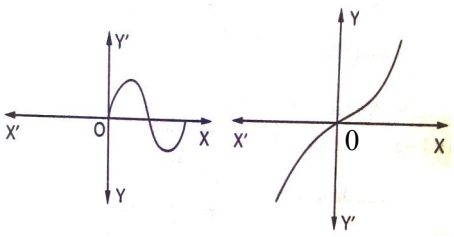
Time Allowed: 180 Minutes **Max. Marks: 100**

NOTE This question paper consists of 29 questions divided into three sections A, B and C. **Section A** comprises of **10 questions** of **one mark** each, **Section B** comprises of **12 questions** of **four marks** each, and **Section C** comprises of **7 questions** of **six marks** each.

[SECTION – A]

Q01. Evaluate: $\int \frac{dx}{x + x\log x}$.

Q02. Which one graph of the Fig.1 represents a function?



(a) Fig.1 (b)

Q03. Write the value of $\int_0^1 \frac{dx}{\sqrt{4x+1}}$.

Q04. Evaluate: $\cos^{-1}\cos 320^\circ - \sin^{-1}\sin 320^\circ$.

Q05. If $A = [a_{ij}] = \begin{pmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{pmatrix}$ and $B = [b_{ij}] = \begin{pmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{pmatrix}$ then, find the value of $a_{22} + b_{21}$.

Q06. If $A = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}$, then find the value of k if $|2A| = k|A|$.

Q07. If $2\hat{i} + 4\hat{j} - \hat{k}$ is perpendicular to $3\hat{i} - m\hat{j} - 2\hat{k}$, then write the value of m.

Q08. If $A = [a_{ij}]$ be a square matrix of order 3 and C_{ij} denotes the cofactor of $[a_{ij}]$ in A. If $|A| = 5$, then write the value of $a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$.

Q09. If a line makes the angles α, β, γ with the coordinate axes, then evaluate the following:

$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma.$$

Q10. Find the value of μ, for which $\mu(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

[SECTION – B]

Q11. If $\log(x^2 + y^2) = 2\tan^{-1}(y/x)$ then, show that $dy/dx = (x + y)/(x - y)$.

(OR) Let $f(x) = x|x|$ for all real values of x. Discuss the derivability of $f(x)$ at $x = 0$.

Q12. Evaluate: $\int \frac{\cos x \; dx}{(2 + \sin x)(3 + 4\sin x)}$.

Q13. Using differentials, find the approximate value of $f(2.01)$, where $f(x) = 4x^3 + 5x^2 + 2$.

(OR) A man of height 1.5m walks away along a straight line from the foot of a light post 6m high at the rate of 3m/sec. (a) How fast does the length of his shadow increase? (b) How fast does the end of his shadow move?

Q14. On the set R–{–1}, a binary operation is defined by a * b = a + b + ab for all a, b ∈ R–{–1}. Prove that * holds both commutative & associative properties on R–{–1}. Find the identity element and prove that every element of R–{–1}is invertible.

Q15. Decompose the vector $6\hat{i} - 3\hat{j} - 6\hat{k}$ into the vectors which respectively are parallel and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$.

(OR) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, show that $\frac{\vec{a}.\vec{b}\times\vec{c}}{\vec{c}.\vec{a}\times\vec{b}} + \frac{\vec{b}.\vec{a}\times\vec{c}}{\vec{c}.\vec{a}\times\vec{b}} = 0$.

Q16. Solve the differential equation: $(x^3 + y^3)dy - x^2y\,dx = 0$.

Q17. Using properties of determinants, prove that: $\begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$.

Q18. Solve for x: $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)+\tan^{-1}\left(\frac{2x}{x^2-1}\right)=\frac{2\pi}{3}$.

(OR) Solve the equation for x: $\sec^{-1}\left(\frac{x}{a}\right)-\sec^{-1}\left(\frac{x}{b}\right)=\sec^{-1}b-\sec^{-1}a$.

Q19. Find the equations of the line(s) passing through the origin (0, 0, 0) which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle of $\pi/3$.

Q20. Evaluate: $\int \cos 2\theta \log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right) d\theta$.

Q21. Obtain the differential equation of all the circles of radius r.

Q22. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive?

[SECTION – C]

Q23. One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as a linear programming problem and solve it graphically. Explain the importance of balanced diet in food.

Q24. Using integration, find the area of the region: $\{(x,y): 9x^2+y^2 \le 36,\ 3x+y \ge 6\}$. What is the importance of integration in life.

Q25. Check if the lines $\frac{x+3}{-3}=(y-1)=\frac{z-5}{5}$ and $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar. If coplanar, then find the equation of plane containing them. Hence find the length of the foot of perpendicular of plane drawn from origin.

Q26. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2R/\sqrt{3}$. Also find the maximum volume.

Q27. Evaluate: $\int_1^4 (|x-1|+|x-2|+|x-4|)\,dx$. **(OR)** Evaluate: $\int x^2\cos^{-1}x\,dx$.

Q28. In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses or copies the answer. Let 1/2 be the probability that he knows the answer, 1/4 be the probability that he guesses and 1/4 that he copies it. Assuming that a student, who copies the answer, will be correct with the probability 3/4, what is the probability that the student knows the answer, given that he answered it correctly?

Aryan does not know the answer to one of the questions in the test. The evaluation process has negative marking. Which value would Aryan violate if he resorts to unfair means? How would an act like the above hamper his character development in the coming years?

(OR) A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

Q29. If $A=\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$ then, find the product AB.

Using this solve the following system of equations:
$3x-4y+2z=-1,\ 2x+3y+5z=7,\ x+z=2$.

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HINTS & ANSWERS *for* PTRS XII – 10 [2012-2013]

Q01. $\log|1 + \log x| + k$

Q02. **Vertical Line Test:** The graph (a) represents the function of x, because vertical line drawn in (a) meets the graph at only one point i.e., for one x in domain there exists only one $f(x)$ in the co-domain.

Q03. $\frac{\sqrt{5}-1}{2}$ Q04. 80^o Q05. 1

Q06. $k = 4$ Q07. 2 Q08. 5 Q09. –1 Q10. $\pm\frac{1}{\sqrt{3}}$

Q11. OR Differentiable at x = 0. Q12. $\frac{1}{5}\log|(3 + 4\sin x)/(2 + \sin x)| + k$

Q13. 54.68 (OR) 1m/sec, 4m/sec

Q14. Identity Element: 0 is the identity element for * defined on R–{–1}. Also, tnverse of an element a is: –a/(a+1) ∈ R–{–1}.

Q15. $-\hat{i}-\hat{j}-\hat{k}, 7\hat{i}-2\hat{j}-5\hat{k}$

Q16. $-\frac{x^3}{3y^3}+\log|y|=k$ Q18. $x=2-\sqrt{3}$ OR $x=\pm ab$

Q19. $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}, \frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$ Q20. $\frac{\sin 2\theta}{2}\log\tan\left(\frac{\pi}{4}+\theta\right)-\frac{1}{2}\log\sec 2\theta + k$

Q21. $[1+(y_1)^2]^3 = r^2(y_2)^2$

Q22. Since a second order determinant has four entries which may be 0 or 1 in present case. Total number of determinants $= 2^4 = 16$. The only positive determinants are $\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix}$, $\begin{vmatrix}1 & 0\\1 & 1\end{vmatrix}$ and $\begin{vmatrix}1 & 1\\0 & 1\end{vmatrix}$. Since each entry of the above determinants can be selected with probability 1/2, therefore the, required probability $= 3\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)=\frac{3}{16}$.

Q23. Maximum number of cakes are: 30; 20 of kind one and 10 cakes of another kind.

Q24. $3(\pi-2)$ sq.units Q25. x – 2y + z = 4, $\frac{2}{3}\sqrt{6}$ units

Q26. $\left(\frac{4\pi R^3}{3\sqrt{3}}\right)$ cubic units Q27. 23/2 OR $\frac{x^3\cos^{-1}x}{3}-\frac{2+x^2}{9}\sqrt{1-x^2}+k$

Q28. Required probability $= \frac{2}{3}$. If Aryan copies the answer, he will be violating the value of honesty in his character. He should not guess the answer as well as that may fetch him negative marking for a wrong guess. He should accept the question the way it is and leave it unanswered as cheating may get him marks in this exam but this habit may not let him develop integrity of character in the long run.

OR Let E_1, E_2 and E be the events defined as 'the person has disease', 'the person is healthy' and 'the test is positive' respectively. So, it is clear that $P(E_1) = 0.1\% = 0.001$, and $P(E_2) = 1 - 0.001 = 0.999$. Also, we have $P(E|E_1) = 99\% = 0.99$, $P(E|E_2) = 0.5\% = 0.005$.

Hence required probability, $P(E_1|E) = 22/133$.

Q29. –9I; $x = 3, y = 2, z = -1$.

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