

AADHARSHILA STUDIES

Sample Paper1 –Mathematics

CLASS – XII

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Time allowed: 3hours

Maximum Marks: 100.

General Instructions:

- (i) All questions are compulsory.
- (ii) The question paper consist of 29 questions divided into three sections A, B and C.
Section A
Comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 07 questions of six marks each.
- (iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each .You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted. You may ask for logarithmic tables , if required.

SECTION – A

1. Show that the binary operation $*$: $R \times R \rightarrow R$ defined by $a * b = a - 2b$ is not commutative.
2. Find the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.
3. For a matrix of order 3 X 3, find the value of K if $|2A| = K|A|$.
4. Find the product $\begin{bmatrix} -5 \\ 3 \\ 2 \end{bmatrix} \begin{pmatrix} 4 & -1 & 3 \end{pmatrix}$.
5. For what value of x, the matrix $\begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$ is a singular matrix?
6. Find the derivative of $\cos^{-1}(\sin x)$ w. r. t. x
7. Evaluate $\int \frac{dx}{\sqrt{x+x}}$
8. Find the direction cosines of the vector $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$.
9. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ find $\vec{a} \cdot \vec{b}$.
10. Write the vector equation of the line whose Cartesian equations are $\frac{x-3}{2} = \frac{y+7}{3} = \frac{z-1}{5}$.

SECTION-B

11. Show that the relation R defined by $(a, b) R (c, d) \Rightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation.

12. Prove that $2 \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$

OR

Express $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$ in the simplest form.

13. Using properties of determinants, show that

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (a^3 - 1)^2$$

14. If $y = \sin(\log x)$, prove that $x^2 \frac{dy}{dx} + x \frac{dy}{dx} + y = 0$.

15. For what value of 'k' is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} 2x - 1; & x < 2 \\ k; & x = 2 \\ 3x - 1; & x > 2 \end{cases}$$

16. Find the equation of the tangent to the curve $x = \sin 3t, y = \cos 2t$ at $t = \frac{\pi}{4}$.

OR

Using differentials, evaluate $\sqrt{0.037}$, up to four decimal places.

17. Evaluate : $\int_0^{\frac{\pi}{2}} \log \sin x dx$

18. Solve the differential equation

$$\frac{dy}{dx} = 1 + e^{x-y}$$

OR

Solve the differential equation

$$\cos^2 x \frac{dy}{dx} + y = \tan x$$

19. Solve the differential equation $x \frac{dy}{dx} + y = x \cos x + \sin x$ given that $y\left(\frac{\pi}{2}\right) = 1$

20. Find the unit vector perpendicular to the plane ABC where position vector of points A, B and C are $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{k}$ respectively.

21. Find the vector equation of a plane passing through the point $(2, 0, -1)$ and perpendicular to the line joining the points $(1, 2, 3)$ and $(3, -1, 6)$

OR

Find the foot of perpendicular from the point (2, 3, 4) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also find the length of the perpendicular segment.

22. Five dice are thrown simultaneously. If the occurrence of an even number on a single die is considered a success, find the probability of getting atmost 3 successes.

SECTION-C

23. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$, find AB . Use the result to solve the

following system of linear equations :

$$2x - y + z = -1$$

$$-x + 2y - z = 4$$

$$x - y + 2z = -3$$

24. Evaluate $\int \sqrt{\cot x} dx$

25. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

OR

Find the points of local maxima/minima for the function

$$f(x) = \sin 2x - x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

26. Using definite integrals, find the area of the triangular region whose vertices are P(1, 0), Q(2, 2) and R(3, 1).
27. There are two bags I and II containing 3 red and 4 white balls and 2 red and 3 white balls respectively. A bag is selected at random and a ball is drawn from it. If it is found to be red ball, find the probability that it is drawn from the first bag.
28. A diet for a sick person must contain at least 4000 units of vitamins, 50 units' oif minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs 5 and Rs 4 per unit respectively. One unit of food A contains 200 units of vitamins, 1 unit of minrals and 40 units of calories, while one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the food A and B should be used to have least cost, but it must satisfy the requirements of the sick person. From the question of LPP and solve it graphically.
29. Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.