

**General Instructions:**

(a) All questions are compulsory.

(b) The question paper consist of **29 questions** divided into **three sections A, B and C**. Section A comprises of **10 questions of one mark** each, section B comprises of **12 questions of four marks** each and section C comprises of **07 questions of six marks** each.

(c) All questions in Section A are to be answered in one word, one sentence or as per **the exact requirement** of the question.

(d) There is no overall choice. However, **internal choice** has been provided in **04 questions of four marks each** and **02 questions of six marks each**. You have to attempt only one of the alternatives in all such questions.

(e) Use of calculators in not permitted. You may ask for logarithmic tables, if required.

**[SECTION - A]**

**Q01.** If \* is a binary operation given by  $*$ :  $R \times R \rightarrow R$ ,  $a * b = a + b^2$ , then what is the value of  $-2*5$ ?

**Q02.** If  $\sin^{-1} : [-1, 1] \rightarrow \left[ \frac{\pi}{2}, \frac{3\pi}{2} \right]$  is a function, then write the value of  $\sin^{-1} \left( -\frac{1}{2} \right)$ .

**Q03.** It is given that  $\begin{pmatrix} 9 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$ . What do you get on applying elementary row transformation  $R_1 \rightarrow R_1 - 2 R_2$  on both the sides?

**Q04.** Write the degree of the differential equation:  $\left[ 1 + \left( \frac{dy}{dx} \right)^3 \right] = \left( \frac{d^2y}{dx^2} \right)^{2/3}$ .

**Q05.** If A and B are square matrices of order 3 such that  $|A| = -1$  and  $|B| = 4$ , then write the value of  $|3 (AB)|$ .

**Q06.** What is the value of  $|\hat{i} - \hat{j}|^2$ ?

**Q07.** What is the distance between the planes  $3x + 4y - 7 = 0$  and  $6x + 8y + 6 = 0$ ?

**Q08.** Evaluate:  $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx$ .

**Q09.** Write the value of integral:  $\int_{-\pi/2}^{\pi/2} (\sin^{83} x + x^{123}) dx$ .

**Q10.** Write the integrating factor for the linear differential equation:  $x \frac{dy}{dx} - y = x^2$ .

**[SECTION - B]**

**Q11.** Discuss the continuity of  $f(x) = |x + 1| + |x + 2|$  at  $x = -2$ .

**Q12.** Determine the vector equation of a line passing through  $(1, 2, -4)$  and perpendicular to the two lines  $\vec{r} = 8\hat{i} - 16\hat{j} + 10\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$  and  $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$ .

**Q13.** Prove that:  $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$ .

**(OR)** Solve the equation:  $\tan^{-1}(2 + x) + \tan^{-1}(2 - x) = \tan^{-1} \frac{2}{3}$ ,  $-\sqrt{3} < x < \sqrt{3}$ .

**Q14.** Find a particular solution of the following differential equation:

$2y e^{\frac{x}{y}} dx + (y - 2x e^{\frac{x}{y}}) dy = 0$  given that  $x = 0$  when  $y = 1$ . **Q15.** Evaluate:  $\int_0^{\pi/6} \sin^4 x \cos^3 x dx$ .

**Q16.** Find the equation of tangent to the curve  $x = \sin 3t$ ,  $y = \cos 2t$  at  $t = \frac{\pi}{4}$ .

**(OR)** Find the intervals in which the function  $f(x) = \sin^4 x + \cos^4 x$ ,  $0 < x < \frac{\pi}{2}$  is strictly increasing or strictly decreasing.

**Q17.** There are three coins. One is a biased coin that comes up with tail 60% of the times, the second is also a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it showed heads. What is the probability that it was the unbiased coin? If one of these biased coins is used for the toss in a cricket match then, which value of life will be demolished?

**Q18.** Evaluate:  $\int x(\log x)^2 dx$ .

**Q19.** On the set  $R - \{-1\}$ , a binary operation is defined by  $a * b = a + b + ab$  for all  $a, b \in R - \{-1\}$ . Prove that  $*$  is commutative on  $R - \{-1\}$ . Find the identity element and prove that every element of the set  $R - \{-1\}$  is invertible.

**(OR)** Let  $n$  be a fixed positive integer and  $R$  be the relation in  $Z$  defined as  $aRb$  if and only if " $a - b$  is divisible by  $n$ ",  $\forall a, b \in Z$ . Show that  $R$  is an equivalence relation.

**Q20.** Find  $\frac{dy}{dx}$ , if  $x = e^\theta \left( \theta + \frac{1}{\theta} \right)$  and  $y = e^{-\theta} \left( \theta - \frac{1}{\theta} \right)$ .

**(OR)** Using mean value theorem, prove that there is a point on the curve  $y = 2x^2 - 5x + 3$  between the points  $A(1, 0)$  and  $B(2, 1)$ , where tangent is parallel to the chord  $AB$ . Also, find that point.

**Q21.** Find the differential equation of all the circles which pass through the origin and whose centres lie on x-axis.

**Q22.** Solve for  $x$ : 
$$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0.$$

### [SECTION - C]

**Q23.** Find the area of the triangle formed by positive x-axis, and the normal and tangent to the circle  $x^2 + y^2 = 4$  at  $(1, \sqrt{3})$ , using integration.

**Q24.** If  $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ , then show that  $A$  satisfies the equation  $A^3 - 4A^2 - 3A + 11I = 0$ . Hence find  $A^{-1}$ .

**Q25.** Four defective bulbs are accidentally mixed with six good ones. If it is not possible to just look at a bulb and tell whether or not it is defective, find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

**Q26.** Find the equation of the plane passing through the intersection of the planes  $x + 3y + 6z = 0$  and  $3x - y - 4z = 0$  and whose perpendicular distance from origin is unity.

**(OR)** Find the distance of  $P(3, 4, 5)$  from the plane  $x + y + z = 2$  measured parallel to the line  $2x = y = z$ .

**Q27.** Evaluate: 
$$\int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x) dx.$$

**Q28.** A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contains at least 8 units of Vitamin A and 10 units of Vitamin C. Food 'I' contains 2units/kg of Vitamin A and 1unit/kg of Vitamin C. Food 'II' contains 1unit/kg of Vitamin A and 2units/kg of Vitamin C. It costs ₹50 per kg to purchase Food 'I' and ₹70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem to minimize the cost of such a mixture and solve it graphically. Justify the importance of balanced diet.

**Q29.** An isosceles triangle of vertical angle  $2\theta$  is inscribed in a circle of radius  $a$ . Show that the area of triangle is maximum when  $\theta = \pi/6$ .

**(OR)** A telephone company in a town has 500 subscribers on its list and collects fixed charges of ₹300/- per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of ₹1/-, one subscriber will discontinue the service. Find what increase will bring the maximum profit?

*Hey, Good Luck  
For Your Exams.!*

