TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

CODE:- AG-X-9999 पजियन क्रमांक **REG.NO:-TMC -D/79/89/36**

General Instructions :

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 5 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

सामान्य निर्देश :

1. सभी प्रश्न अनिवार्य हैं।
2. इस प्रश्न पत्र में 29 प्रश्न है, जो 3 खण्डों में अ, ब, व स है। खण्ड – अ में 10 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड – ब में 12 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको के हैं। खण्ड – स में 7 प्रश्न हैं और प्रत्येक प्रश्न 6 अंको का है।
3. प्रश्न संख्या 1 से 10 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
4. इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 4 प्रश्न 4 अंको में और 2 प्रश्न 6 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
5. कैलकुलेटर का प्रयोग वर्जित हैं ।
6. कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ 5 हैं।
7. प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें।

Pre-Board Examination 2011 -12

Time : 3 Hours	अधिकतम समय : 3
Maximum Marks : 100	अधिकतम अंक : 100
Total No. Of Pages :5	कुल पृष्ठों की संख्या : 5

CLASS – XII CBSE MATHEMATICS

SECTION A

NOTE:- Choose the correct answer from the given four options in each of the Questions 1 to 3.

Q.1	If $*$ is a binary operation given by $*: \mathbf{R} \times \mathbf{R} \to \mathbf{R}$, $a * b = a + b^2$, then $-2*5$ is (A) -52 (B) 23 (C) 64 (D) 13	ANS : **B**
Q.2	If $\sin^{-1} : [-1, 1] \to \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is a function, then value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is (A) $\frac{-\pi}{6}$ (B) $\frac{-\pi}{6}$ (C) $\frac{5\pi}{6}$ (D) $\frac{7\pi}{6}$	ANS : D
Q.3	Given that $\begin{pmatrix} 9 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$. Applying elementary row transformation $R_1 \to R_1 - 2R_2$ on both sides, we get (A) $\begin{pmatrix} 3 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -4 \\ 1 & 2 \end{pmatrix}$ (B) $\begin{pmatrix} 3 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$ (C) $\begin{pmatrix} -3 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ -3 & 2 \end{pmatrix}$ (D) $\begin{pmatrix} -3 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$	ANS : **B**
Q.4	If A is a square matrix of order 3 and $|A| = 5$, then what is the value of $|Adj.\ A|$?	ANS :25
Q.5	If A and B are square matrices of order 3 such that $|A| = -1$ and $|B| = 4$, then what is the value of $|3(AB)|$? ANS : - 108	
NOTE:-	Fill in the blanks in each of the Questions 6 to 8 .	

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Q.6	The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^3\right]=\left(\frac{d^2y}{dx^2}\right)^2$ is______. ANS : 2
Q.7	The integrating factor for solving the linear differential equation $x\frac{dy}{dx}-y=x^2$ is_______. ANS : 1/ x
Q.8	The value of $\left\|\hat{i}-\hat{j}\right\|^2$ is _______. ANS : 2
Q.9	What is the distance between the planes $3x + 4y - 7 = 0$ and $6x + 8y + 6 = 0$? ANS : 2 unit
Q.10	If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=99$, then what is the value of $\|\vec{x}\|$? ANS : 10
	SECTION B
Q.11	Let n be a fixed positive integer and R be the relation in **Z** defined as a R b if and only if $a - b$ is divisible by n, $\forall\ a, b \in Z$. . Show that R is an equivalence relation. Ans : (i) Since a R a, $\forall\ a \in Z$, and because 0 is divisible by n, therefore R is reflexive. 1 (ii) a R $b \Rightarrow a - b$ is divisible by n, then $b - a$, is divisible by n, so b R a. Hence R is symmetric. 1 (iii) Let a R b and b R c, for $a,b,c, \in$ **Z**. Then $a - b = n\,p$, $b - c = n\,q$, for some $p, q \in$ **Z** Therefore, $a - c = n\ (p + q)$ and so a R c. 1 Hence R is reflexive and so equivalence relation. 1
Q.12	Prove that $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$. ans: LHS $= \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18}$ $= \tan^{-1}\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}.\frac{1}{8}} + \tan^{-1}\frac{1}{18} = \tan^{-1}\left(\frac{15}{55}\right) + \tan^{-1}\frac{1}{18}$ $= \tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18} = \tan^{-1}\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}\ \frac{1}{18}} = \tan^{-1}\frac{65}{195}$ $= \tan^{-1}\frac{1}{3} = \cot^{-1}3$ = RHS OR Solve the equation $\tan^{-1}(2+x)+\tan^{-1}(2-x)=\tan^{-1}\frac{2}{3}, -\sqrt{3}>x>\sqrt{3}$. ans : Since $\tan^{-1}(2 + x) + \tan^{-1}(2 - x) = \tan^{-1}\frac{2}{3}$ Therefore, $\tan^{-1}\frac{(2+x)+(2-x)}{1-(2+x)\ (2-x)} = \tan^{-1}\frac{2}{3}$ Thus $\frac{4}{x^2-3}=\frac{2}{3}$ $\Rightarrow\ x^2 = 9\ \Rightarrow x = \pm 3$

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Q.13	Solve for x, $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$ .ans Given, $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$ Using $\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$, we get $\begin{vmatrix} x+2 & x+6 & x-1 \\ 4 & -7 & 3 \\ -3 & -4 & 7 \end{vmatrix} = 0$ Using $\begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix}$, we get $\begin{vmatrix} x+2 & 4 & -3 \\ 4 & -11 & -1 \\ -3 & -1 & 10 \end{vmatrix} = 0$ Therefore, $(x+2)(-111) - 4(37) - 3(-37) = 0$ $x = -\frac{7}{3}$ OR If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & -3 \end{pmatrix}$, verify that $(AB)' = B'A'$. ans : $AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & -3 \end{pmatrix} = \begin{pmatrix} 7 & 3 & -4 \\ 15 & 5 & -6 \end{pmatrix}$ Therefore, $LHS = (AB)' = \begin{pmatrix} 7 & 15 \\ 3 & 5 \\ -4 & -6 \end{pmatrix}$ $RHS = B'A' = \begin{pmatrix} 1 & 3 \\ -1 & 2 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 15 \\ 3 & 5 \\ -4 & -6 \end{pmatrix}$ and hence LHS = RHS
Q.14	Determine the value of k so that the function: $f(x) = \begin{cases} \frac{k.\cos 2x}{\pi - 4x}, & \text{if } x \neq \frac{\pi}{4} \\ 5, & \text{if } x = \frac{\pi}{4} \end{cases}$ is continuous at $x = \frac{\pi}{4}$. ans : Since f is continous at $x = \frac{\pi}{4}$, we have $\lim_{x \to \frac{\pi}{4}} f(x) = 5$. Now $\lim_{x \to \frac{\pi}{4}} f(x) = \lim_{x \to \frac{\pi}{4}} \frac{k.\cos 2x}{\pi - 4x} = \lim_{y \to 0} \frac{k \cos 2(\frac{\pi}{4} - y)}{\pi - 4(\frac{\pi}{4} - y)}$, where $\frac{\pi}{4} - x = y$, $= \lim_{y \to 0} \frac{k.\cos(\frac{\pi}{2} - 2y)}{\pi - \pi + 4y} = \lim_{y \to 0} \frac{(k \sin 2y)}{2.\ 2y} = \frac{k}{2}$ Therefore, $\frac{k}{2} = 5 \Rightarrow k = 10$.
Q.15	If $y = e^{a\cos^{-1}x}$, show that $(1-x^2)\frac{d^2y}{d^2x} - x\frac{dy}{dx} - a^2y = 0$. ans: $y = e^{a\cos^{-1}x} \Rightarrow \frac{dy}{dx} = e^{a\cos^{-1}x} \frac{(-a)}{\sqrt{1-x^2}}$ Therefore, $\sqrt{1-x^2}\frac{dy}{dx} = -ay$.......(1) Differentiating again w.r.t. x, we get $\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} = -\frac{ady}{dx}$ $= -a(-ay)$ [from 1] $\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -a\sqrt{1-x^2}\frac{dy}{dx}$ Hence $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$.
Q.16	Find the equation of the tangent to the curve $x = \sin 3t$, $y = \cos 2t$ at $t = \frac{\pi}{4}$ .ans : $\frac{dx}{dt} = +3\cos 3t$, $\frac{dy}{dt} = -2\sin 2t$ Therefore, $\frac{dy}{dx} = -\frac{2\sin 2t}{3\cos 3t}$, and $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{-2\sin\frac{\pi}{2}}{3\cos 3\frac{\pi}{4}} = \frac{-2}{3.(-\frac{1}{\sqrt{2}})} = \frac{2\sqrt{2}}{3}$

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Also $x = \sin 3t = \sin 3\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $y = \cos 2t = \cos\frac{\pi}{2} = 0.$ Therefore, Point is $\left(\frac{1}{\sqrt{2}}, 0\right)$

Hence, equation of tangent is $y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$ Therefore, $2\sqrt{2}\,x - 3y - 2 = 0$

OR

Find the intervals in which the function $f(x) = \sin^4 x + \cos^4 x,\ 0 < x < \frac{\pi}{2}$, ans :

$f'(x) = 4\sin^3 x\ \cos x - 4\cos^3 x\ \sin x$

$= -4\sin x\cos x\,(\cos^2 x - \sin^2 x)$

$= -\sin 4x$. Therefore,

$f'(x) = 0 \Rightarrow 4x = n\pi \Rightarrow x = n\frac{\pi}{4}$

Now, for $0 < x < \frac{\pi}{4}$,

$f'(x) < 0$

Therefore, f is strictly decreasing in $\left(0, \frac{\pi}{4}\right)$ Similarly, we can show that f is strictly increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Q.17

Evaluate $\int_0^{\frac{\pi}{6}} \sin^4 x\cos^3 x\,dx$.

Ans $I = \int_0^{\frac{\pi}{6}} \sin^4 x\cos^3 x\,dx = \int_0^{\frac{\pi}{6}} \sin^4 x\,(1 - \sin^2 x)\cos x\,dx$

$= \int_0^{\frac{1}{2}} t^4(1 - t^2)\,dt$, where $\sin x = t$ $= \int_0^{\frac{1}{2}} (t^4 - t^6)\,dt = \left[\frac{t^5}{5} - \frac{t^7}{7}\right]_0^{\frac{1}{2}}$ $= \frac{1}{5}\left(\frac{1}{2}\right)^5 - \frac{1}{7}\left(\frac{1}{2}\right)^7 = \frac{1}{32}\left(\frac{1}{5} - \frac{1}{28}\right) = \frac{23}{4480}$

Q.18

Evaluate $\int \frac{3x+1}{2x^2 - 2x + 3}dx$.

Ans: $I = \int \frac{3x+1}{2x^2 - 2x + 3}dx = \int \frac{\frac{3}{4}(4x-2) + \frac{5}{2}}{2x^2 - 2x + 3}dx$

$= \frac{3}{4}\int \frac{4x-2}{2x^2 - 2x + 3}dx + \frac{5}{4}\int \frac{1}{x^2 - x + \frac{3}{2}}dx$ $= \frac{3}{4}\log|2x^2 - 2x + 3| + \frac{5}{4}\frac{2}{\sqrt{5}}\tan^{-1}\frac{2x-1}{\sqrt{5}} + c$

$= \frac{3}{4}\log|2x^2 - 2x + 3| + \frac{\sqrt{5}}{2}\tan^{-1}\frac{2x-1}{\sqrt{5}} + c$

OR

Evaluate $\int x.(\log x)^2\,dx$ .ans :

$I = \int x(\log x)^2 .dx = \int (\log x)^2\,x\,dx$ $= (\log x)^2\frac{x^2}{2} - \int 2\log x\frac{1}{x}\frac{x^2}{2}dx$ $= \frac{x^2}{2}(\log x)^2 - \int \log x.\,x\ dx$

$= \frac{x^2}{2}(\log x)^2 - \left[\log x.\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}\,dx\right]$ $= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\ \log x + \frac{x^2}{4} + c$

Q.19

Find a particular solution of the differential equation: $2y\,e^{\frac{x}{y}}\,dx + (y - 2x\,e^{\frac{x}{y}})\,dy = 0$, given that $x = 0$ when $y = 1$.

ans: Given differential equation can be written as $\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}} - y}{2y.e^{\frac{x}{y}}}$

Putting $\frac{x}{y} = v \Rightarrow x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ Therefore, $v + y\frac{dv}{dy} = \frac{2vye^v - y}{2ye^v} = \frac{2ve^v - 1}{2e^v}$

$y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v$ Hence $2e^v dv = -\frac{dy}{y}$ $\Rightarrow 2e^v = -\log|y| + c$ or $2e^{\frac{x}{y}} = -\log|y| + c$ when $x = 0$, $y = 1 \Rightarrow C = 2$ Therefore, the particular solution is $2e^{\frac{x}{y}} = -\log|y| + 2$

Q.20

If $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, then find the projection of $\vec{b} + \vec{c}$ along $\vec{a}$. ans

$\vec{b} + \vec{c} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (2\hat{i} - \hat{j} + 4\hat{k}) = 3\hat{i} + \hat{j} + \hat{k}$ $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$

Projection of $(\vec{b} + \vec{c})$ along $\vec{a} = \frac{(\vec{b} + \vec{c}).\vec{a}}{|\vec{a}|}$ is $\frac{6 - 2 + 1}{\sqrt{4 + 4 + 1}} = \frac{5}{3}$ units

Q.21

Determine the vector equation of a line passing through (1, 2, –4) and perpendicular to the two lines $\vec{r} = (8\hat{i} - 16\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ & $(15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$. ans: A vector perpendicular to the two lines is given as $(3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$

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$= 24\hat{i} + 36\hat{j} + 72\hat{k}$ or $12(2\hat{i} + 3\hat{j} + 6\hat{k})$ Therefore, Equation of required line is $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

Q.22 There are three coins. One is a biased coin that comes up with tail 60% of the times, the second is also a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it showed heads. What is the probability that it was the unbiased coin? Ans : Let E_1:selection of first (biased) coin ;E_2: selection of second (biased) coin ;E_3: selection of third (unbiased) coin

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ Let A denote the event of getting a head

Therefore, $P\left(\frac{A}{E_1}\right) = \frac{40}{100}$, $P\left(\frac{A}{E_2}\right) = \frac{75}{100}$, $P\left(\frac{A}{E_3}\right) = \frac{1}{2}$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)} = \frac{\frac{1}{3}\cdot\frac{1}{2}}{\frac{1}{3}\cdot\frac{40}{100} + \frac{1}{3}\cdot\frac{75}{100} + \frac{1}{3}\cdot\frac{1}{2}} = \frac{10}{33}$$

SECTION C

Q.23 Find A^{-1}, where $A = \begin{pmatrix} 4 & 1 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & -2 \end{pmatrix}$. Hence solve the following system of equations $4x + 2y + 3z = 2$, $x + y + z = 1$, $3x + y - 2z = 5$,

ans: $|A| = 4(-3) - 1(-7) + 3(-1) = -12 + 7 - 3 = -8$

Therefore, $A^{-1} = -\frac{1}{8}\begin{pmatrix} -3 & 5 & -2 \\ 7 & -17 & 2 \\ -1 & -1 & 2 \end{pmatrix}$ Given equations can be written as

$$\begin{pmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

$\Rightarrow A'.X = B \Rightarrow X = (A'^{-1})B = (A^{-1})'B$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{8}\begin{pmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

$$= -\frac{1}{8}\begin{pmatrix} -6 & +7 & -5 = & -4 \\ 10 & -17 & -5 = & -12 \\ -4 & +2 & +10 = & 8 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \\ -1 \end{pmatrix}$$

Therefore, $x = \frac{1}{2}$, $y = \frac{3}{2}$, $z = -1$

OR

Using elementary transformations, find A^{-1}, where $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$. ans

Writing $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$;

$R_2 \to R_2 + R_1 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$

$R_2 \to R_2 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} A$ $R_3 \to R_3 + 2R_2 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$

$R_1 \to R_1 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 4 & 10 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$ $R_1 \to R_1 - 2R_2 \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$

$\Rightarrow A^{-1} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$

Q.24 Show that the semi-vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$. ans :

. Volume $v = v = \frac{1}{3}\pi r^2 h$

$l^2 = h^2 + r^2$

$v = \frac{1}{3}\pi\ (l^2 - h^2)\ h = \frac{1}{3}\pi\ (l^2 h - h^3)$

$\frac{dv}{dh} = \frac{\pi}{3}(l^2 - 3h^2) = 0$

$l = \sqrt{3}h,\quad r = \sqrt{2}h$

$\tan\alpha = \frac{r}{h} = \sqrt{2}$

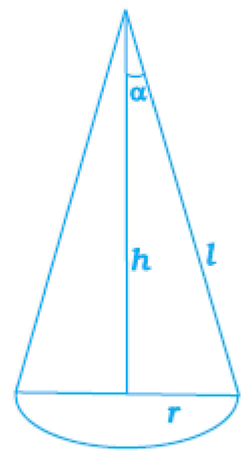

Fig. 2.1

$\alpha = \tan^{-1}\sqrt{2}$

$\frac{d^2 v}{dh^2} = -2\pi h < 0$

Therefore, v is maximum

Q.25 Evaluate $\int_1^3 (3x^2 + 2x + 5)\, dx$ by the method of limit of sum. ans :

$\mathrm{I} = \int_1^3 (3x^2 + 2x + 5)\, dx = \int_1^3 f(x)\, dx$

$= \lim_{h \to o} h\left[f(1) + f(1+h) + f(1+2h) + \ldots\ldots + f(1+(n-1)h)\right] \ldots\ldots$ (i)

where $h = \frac{3-1}{n} = \frac{2}{n}$

$f(1) = 3 + 2 + 5 = 10$

$f(1+h) = 3 + 3h^2 + 6h + 2 + 2h + 5 = 10 + 8h + 3h^2$

$f(1+2h) = 3 + 12h^2 + 12h + 2 + 4h + 5 = 10 + 8.2.h + 3.2^2.h^2$

$f(1+(n-1)h) = 10 + 8(n-1)\,h + 3(n-1)^2.h^2$

$\mathrm{I} = \lim_{n\to\infty} h\left[10n + 8h\frac{n(n-1)}{2} + 3h^2\frac{n(n-1)(2n-1)}{6}\right]$

$= \lim_{n\to\infty}\frac{2}{n}\left[10n + \frac{16}{n}\frac{n(n-1)}{2} + \frac{12}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$

$= \lim_{n\to\infty}\frac{2}{n}\left[10n + 8(n-1)\frac{2}{n}(n-1)\,(2n-1)\right]$

$= \lim_{n\to\infty} 2\left[10 + 8(1 - \frac{1}{n}) + 2(1 - \frac{1}{n})\,(2 - \frac{1}{n})\right]$

$= 2\,[10 + 8 + 4] = 44$

Q.26 Find the area of the triangle formed by positive x-axis, and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$, using integration. Ans :

Equation of tangent to $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

$x + \sqrt{3}y = 4$. Therefore, $y = \frac{4-x}{\sqrt{3}}$

Equation of normal $y = \sqrt{3}x$

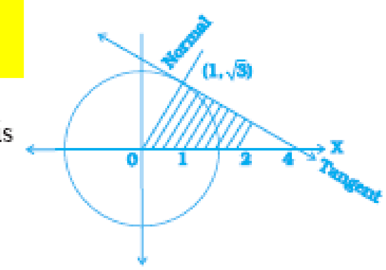

Fig 2.2

Therefore, required area $= \int_0^1 \sqrt{3}\,x\, dx + \int_1^4 \frac{4-x}{\sqrt{3}}\, dx$

$= \left(\sqrt{3}\frac{x^2}{2}\right)_0^1 + \frac{1}{\sqrt{3}}\left(4x - \frac{x^2}{2}\right)_1^4$

$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}\left[8 - \frac{7}{2}\right] = \frac{\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} = 2\sqrt{3}$ sq. units

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Q.27 Find the equation of the plane through the intersection of the planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$ and whose perpendicular distance from origin is unity. Ans : Equation of required plane is $(x + 3y + 6) + \lambda(3x - y - 4z) = 0$. $\Rightarrow \quad (1 + 3\lambda)x + (3 - \lambda)y - 4\lambda z + 6 = 0$ Perpendicular distance to the plane from origin is

Therefore, $\dfrac{6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}} = 1$ or $36 = 1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2$

or $26\lambda^2 = 26 \Rightarrow \lambda = \pm 1$

Equations of required planes are $4x + 2y - 4z + 6 = 0$ and $-2x + 4y + 4z + 6 = 0$ or $2x + y - 2z + 3 = 0$ and $x - 2y - 2z - 3 = 0$

OR

Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$. *ans* : the coordinates of Q(1, 0, 1) . Therefore, $PQ = \sqrt{4+16+16} = 6$ units

Q.28 Four defective bulbs are accidently mixed with six good ones. If it is not possible to just look at a bulb and tell whether or not it is defective, find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot. Ans : Let x denotes the number of defective bulbs

$P(X=0) = \dfrac{{}^6C_4}{{}^{10}C_4} = \dfrac{6.5.4.3}{10.9.8.7} = \dfrac{1}{14}$ $\quad P(X=1) = \dfrac{{}^6C_3\,{}^4C_1}{{}^{10}C_4} = \dfrac{6.5.4.}{10.9.8.7}4 = \dfrac{8}{21}$

$P(X=2) = \dfrac{{}^6C_2\,{}^6C_2}{{}^{10}C_4} = \dfrac{6.5.4.3}{10.9.8.7}.6 = \dfrac{3}{7}$ $\quad P(X=3) = \dfrac{{}^6C_1\,{}^6C_3}{{}^{10}C_4} = \dfrac{6.4.3.2}{10.9.8.7}.4 = \dfrac{4}{35}$

$P(X=4) = \dfrac{{}^4C_4}{{}^{10}C_4} = \dfrac{4.3.2.1}{10.9.8.7} = \dfrac{1}{210}$

X :	0	1	2	3	4
P (X) :	$\frac{1}{14}$	$\frac{8}{21}$	$\frac{3}{7}$	$\frac{4}{35}$	$\frac{1}{210}$

Q.29 A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs 30 while by selling one table the profit is Rs 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problems as a L.P.P. and solve it graphically. Ans :

Let number of chairs to be made per week be x and tables be y

Thus we have to maximise $P = 30x + 60y$

Subject to
$$\left.\begin{aligned} 2x + y &\le 70 \\ x + y &\le 40 \\ x + 3y &\le 90 \\ x \ge 0 \quad & y \ge 0 \end{aligned}\right]$$

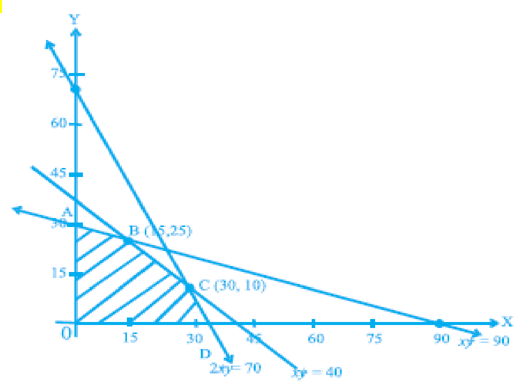

Fig. 2.4

Vertices of feasible region are

A (0,30), B (15, 25), C (30,10), D (35, 0)

P (at A) = 30 (60) = 1800

P (at B) = 30 (15 + 50) = 1950

P (at C) = 30 (30 + 20) = 1500

P (at D) = 30 (35) = 1050

P is Maximum for 15 chairs and 25 tables.

____________x____________

A MAN WHO DOESN'T TRUST HIMSELF ; CAN NEVER TRULY TRUST ANYONE ELSE

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