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TARGET MATHEMATICS THE EXCELLENCE KEY AGYAT GUPTA (M.Sc., M.Phil.)



<u> CODE:- AG-TS-10-3636</u>

पजियन क्रमांक

REGNO:-TMC-D/79/89/36

GENERAL INSTRUCTIONS:

- 1. All questions are compulsory.
- 2. The question paper consists of 34 questions divided into four sections A,B,C and D. Section A comprises of 8 question of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 10 questions of 3 marks each and Section D comprises of 10 questions of 4 marks each.
- 3. Question numbers 1 to 8 in Sections A are multiple choice questions where you are to select one correct option out of the given four.
- 4. There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four mark each. You have to attempt only one If the alternatives in all such questions.
- 5. Use of calculator is not permitted.

सामान्य निर्देश :

- 1. सभी प्रश्न अनिवार्य हैं।
- 2. इस प्रश्न पत्र में 34 प्रश्न है, जो चार खण्डों में अ, ब, स व द में विभाजित है। खण्ड अ में 8 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड ब में 6 प्रश्न हैं और प्रत्येक प्रश्न 2 अंको के हैं। खण्ड स में 10 प्रश्न हैं और प्रत्येक प्रश्न 3 अंको का है। खण्ड द में 10 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको का है।
- 3. प्रश्न संख्या 1 से 8 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
- 4. इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 1 प्रश्न 2 अंको में, 3 प्रश्न 3 अंको में और 2 प्रश्न 4 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
- 5. कैलकुलेटर का प्रयोग वर्जित है।
- 6. इस प्रश्न-पत्र को पढ़ने के लिऐ 15 मिनिट का समय दिया गया है। इस अवधि के दौरान छात्र केवल प्रश्न-पत्र को पढेंगे और वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगें।

PRE-BOARD EXAMINATION 2012 -13

М	A THEMATICS CLASS X	(SA-2)			
Time	$: 3 \text{ to } 3 \frac{1}{4} \text{ Hours}$	अधिकतम समय : 3 से $3\frac{1}{4}$			
Maximum Marks : 90		अधिकतम अंक : 90			
Total	No. Of Pages : 4	कुल पृष्ठों की संख्या : 4			
SECTION A					
Q.1	From the top of a lighthouse 60 metres high with its base a depression of a boat is 30° . The distance of the boat from t (a) $10\sqrt{3}$ m (b) $15\sqrt{3}$ m (c) $20\sqrt{3}$ m (d) none of these	he foot of the lighthouse is			
Q.2	In what ratio does the point $\left(\frac{11}{6}, \frac{17}{6}\right)$ divide the join of A (1 B) 7:5 (C) 2:3 (D) NONE Ans A	, 2) and B(3, 4). (A) 5:7 (
Q.3	Find the probability that a number selected at random from prime. (A) 9/23 (B) 8/25 (C) 8/23 (D) NONE Ans C	the numbers 3, 4, 5,, 25 is			
Q.4	The sum of all three digit numbers which are divisible by 7 (A) 7336 (B) 70336 (C) 128 (D) NONE Ans B	7			
Q.5	If tangents PA and PB from a point P to a circle with central at angle of 80°, then ∠POA is equal to (A) 50° (B) 60° (C) 70° (D) NONE Ans A	e O are inclined to each other			

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	The angle subtended at the centre of a circle of radius 7 cm, by an arc of length 11 cm?				
Q.7	(A) 90 (B) 80 (C) 60 (D) NONE Ans A A right ΔABC right angled at A drawn to circumscribe a circle of radius 5cm with centre O. If AC = 17cm and AB = 18cm, then OC is equal to (a) 10cm (b) 9cm (c) 12cm (d) 13cm Ans d				
Q.8	What is the probability in non leap year that two friends have different birthdays?				
	(A) 1/365 (B) 364/365 (C) 364 / 366 (D) NONE Ans B				
	SECTION B				
Q.9	Using quadratic formula, solve the following equation for $x : abx^2 + (b^2 - ac)x - bc = 0$. Ans c/b , $-b/a$				
Q.10	Two concentric circles are of radii 5cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. Ans 8 cm				
Q.11	Find the arithmetic progression whose third term is 16 and seventh term exceeds its fifth term by 12. Ans $a = 4$; $d = 6$ AP: 4, 10, 16,				
Q.12	Solid cylinder of brass 8 m high and 4 m diameter is melted and recast into a cone of diameter 3 m. Find the height of the cone. Ans 42.66 m				
Q.13	AB is a diameter and AC is a chord of a circle such that $\angle BAC = 30^{\circ}$. If tangent at C				
	intersects AB produced at D, prove that BC=BD.				
Q.14	The length of minute hand of a clock is 14cm. find the area swept by the minute hand in				
	5 minutes. Ans 51.33cm ²				
	OR				
	Rectangular courts are 20 m x 10 m. Find the area of the shaded region Ans 92.5 cm				
Q.15	Ans 92.5 cm				
Q.15	SECTION C The shadow of a flagstaff is three times as long as the shadow of the flagstaff when the sun rays meet the ground at an angle of 60°. Find the angle between the sun rays and the				
Q.15	SECTION C SECTION C The shadow of a flagstaff is three times as long as the shadow of the flagstaff when the sun rays meet the ground at an angle of 60° . Find the angle between the sun rays and the ground at the time of longer shadow. Ans $\theta = 30^{\circ}$ OR The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60° . At a point Y, 40 m vertically above X, the angle of elevation is 45° . Find the height of the tower PQ and the distance XQ. Ans: Height of Tower PQ = 54 . $64 + 40$				
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Q.16	SECTION C The shadow of a flagstaff is three times as long as the shadow of the flagstaff when the sun rays meet the ground at an angle of 60° . Find the angle between the sun rays and the ground at the time of longer shadow. Ans $\theta = 30^{\circ}$ OR The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60° . At a point Y, 40 m vertically above X, the angle of elevation is 45° . Find the height of the tower PQ and the distance XQ. Ans: Height of Tower PQ = 54 . $64 + 40$ = $94 \cdot 64$ & Distance $XQ = \frac{94 \cdot 64 \times 2}{\sqrt{3}} = 109 \cdot 3m$ If I walked $1km/hr$ faster, I would have taken 15 minutes less to walk 3 km. find the rate of my walking. Ans: $\frac{3}{x} - \frac{3}{x+1} = \frac{15}{60} = 3km/h$				
Q.16	SECTION C The shadow of a flagstaff is three times as long as the shadow of the flagstaff when the sun rays meet the ground at an angle of 60° . Find the angle between the sun rays and the ground at the time of longer shadow. Ans $\theta = 30^{\circ}$ OR The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60° . At a point Y, 40 m vertically above X, the angle of elevation is 45° . Find the height of the tower PQ and the distance XQ. Ans: Height of Tower PQ = 54 . $64 + 40$ = $94 \cdot 64$ & Distance $XQ = \frac{94 \cdot 64 \times 2}{\sqrt{3}} = 109 \cdot 3m$ If I walked $1km/hr$ faster, I would have taken 15 minutes less to walk 3 km. find the rate of my walking. Ans: $\frac{3}{x} - \frac{3}{x+1} = \frac{15}{60}$ 3km/h If the point C(-1,2) divides line segment AB in the ratio 3:4, where the co-ordinates of				

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is no male child in the family © there is at least one male child in the family.	Ans. (a	a)

3/8 (b) 1/8 (c) 7/8

Q.19 In fig.,
$$\angle BAD = 78^{\circ}$$
, $\angle DCF = x^{\circ}$ and $\angle DEF = y^{\circ}$. Find the values of x and y.

Q.20 A sum of 7260 is paid off in 20 installments such that each installment is 20 more than preceding installment. Calculate the value of first installment. ANS: Here common difference d = Rs. 20

Number of instalment n = 20

$$S_n = Rs. 7260$$

$$S_n = \frac{n}{2} [2a + (n-1) d]$$

$$7260 = \frac{20}{2} [2a + (20 - 1)20]$$

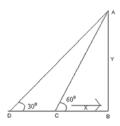
$$\Rightarrow$$
 a = 173

The value of 1st instalment = Rs. 173

- Q.21 Find the coordinates of the points which divide the line segment joining the points (-8, 0) and (4,-8) in four equal parts. Ans (-5,-2),(-2,-4),(1,-6)
- Q.22 Find K if the given value of x is the K th term of the given A.P.

$$5\frac{1}{2}$$
, 11, 16 $\frac{1}{2}$, 22,, $x = 550$. Ans: $k = 100$

 $5\frac{1}{2}$, 11, 16 $\frac{1}{2}$, 22,, x = 550. Ans: k = 100A person standing on the bank of a river observes that the angle of elevation of the top Q.23 of a tree standing on the opposite bank is 60° . When he moves 40m away from the bank, he finds the angle of elevation to be 30° . Find the height of the tree and the width of the river. Solution:- Let height of the tree be y and width of the river be x. CD = 40m



In $\triangle ABD$: tan $30^0 = AB/BD$

$$\frac{1}{\sqrt{3}} = \frac{y}{x+40} \Rightarrow x+40 = y\sqrt{3} - - - - (i)$$

In
$$\triangle ABC$$
 : $\tan 60^{\circ} = AB/BC$: $\sqrt{3} = \frac{y}{x} \Rightarrow y = x\sqrt{3} - - - - - (ii)$

Putting value of y from (ii) to (i)

$$x + 40 = \left(x\sqrt{3}\right)\sqrt{3} = 3x$$

$$40 = 2x$$

$$\Rightarrow x = 20m$$

$$y = x\sqrt{3} = 20\sqrt{3}$$

$$y = x\sqrt{3} = 20\sqrt{3}$$
 . Height of the tree = $20\sqrt{3}m$ & Width of the river =

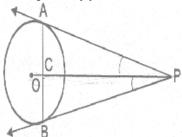
20m.

OR

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There is a small island in between a river 100 meters wide. A tall tree stands on the island P and Q are points directly opposite to each other on the two banks and in line with the tree. If the angles of elevation of the top of the tree from P and Q are 30° and 45° respectively, find the height of tree. Ans $50(\sqrt{3}+1)=36.6$

Q.24 Prove that the tangents at the extremities of any chord make equal angles with the chord. **Sol.** Let AB be a chord of a circle with centre O, and let AP and BP be the tangents at A and B respectively. Suppose, the tangents meet at point P. Join OP.



Suppose OP meets AB at C.

We have to prove that

∠PAC = ∠PBC

In triangles PCA and PCB

PA = PB [.. Tangent from an external point are equal]

∠APC = ∠BPC [∴ PA and PB are equally inclined to OP]

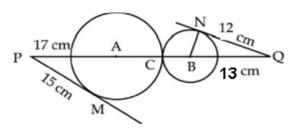
And PC = PC [Common]

So, by SAS criteria of congruence

 $\Delta PAC \cong \Delta BPC$

OR

In fig. two circles with center A and B touch each externally. PM = 15 cm is tangent to circle with center A and QN = 13 cm is tangent to circle with center B from external point Q. if PA = 17 cm and BQ = 12cm. find the distance between the centers A and B of circles.



ANS:

Now AM =
$$\sqrt{17^2 - 15^2}$$

$$= 8$$

$$AC = AM = 8 \text{ cm}$$

$$Again BN = \sqrt{13^2 - 12^2}$$

$$=5$$

$$\therefore$$
 BC = BN = 5 cm

$$\therefore AB = AC + CB$$

$$=8+5=13$$
 cm

SECTION D

- Q.25 One fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to the mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels. Ans x = 36, y = 6, y = -10/3
- Q.26 Draw a triangle ABC with side BC = 7cm, $\angle B = 45^{\circ}$, $\angle A = 105^{\circ}$, then construct a triangle whose sides are $\frac{5}{3}$ times the corresponding side of $\triangle ABC$.

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Q.27 A well with 10m inside diameter is dug 14 m deep. Earth taken out of it is spread all a round to a width of 5 m to form an embankment. Find the height of embankment.

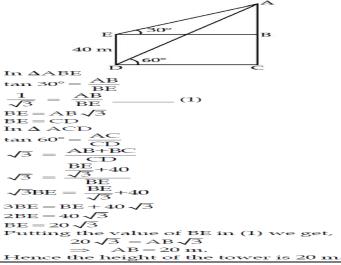
$$\frac{7700}{22 \times 75} = 4.66 \, m$$

OR

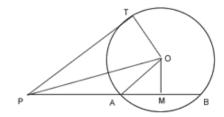
A hemispherical tank of radius $_{1\frac{3}{4}}$ m is full of water. It is connected with a pipe which empties it at the rate of 7 litres per second. How much time will it take to empty the tank completely? Ans 1601.5 sec OR 26.6 MINUTE

- Q.28 Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
- Q.29 The sum of three numbers in A.P. is 27 and their product is 648. Find the numbers.

 Ans: 6, 912
- Q.30 The angle of elevation of the top of a vertical tower from a point on the ground is 60°. At a point 40m vertically above the first point of observation, the angle of elevation is 30°. Find the height of the tower. ANS: Let the height of the tower be AC



Q.31 If PAB is a secant to a circle intersecting it at A and B and PT is a tangent then PA.PB = PT².



Given: - PAB is secant intersecting the circle with centre O at A and B and a tangent PT at T.

To Prove: - PA.PB = PT²

Construction: - $OM \perp AB$ is drawn OA, OP and OT are joined.

Proof: -PA = PM - AM

$$PB = PM + MB$$

$$= PM = AM$$
 (: $AM = MB$)

$$\therefore PA.PB = (PM - AM) \cdot (PM + AM)$$
$$= PM^{2} - AM^{2}$$

Also $OM \perp AB = PM^2 = OP^2 - OM^2$ [Pythagoras theo.]

and $AM^2 = OA^2 - OM^2$ [Pythagoras theo.]

$$PA.PB = PM^{2} - AM^{2}$$

$$= (OP^{2} - OM^{2}) - (OA^{2} - OM^{2})$$

$$= OP^{2} - OM^{2} - OA^{2} + OM^{2}$$

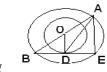
$$= OP^{2} - OA^{2}$$

$$= OP^{2} - OT^{2}$$
[∴ OA = OT radii]

· PA.PB = PT² [Pythagoras theo.]

OR

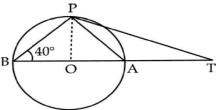
The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle BD is tangent to the smaller circle touching it at D .Find the length of AD in



given fig

. Length of AD = $\sqrt{361} = 19cm$

Q.32 In the figure, BOA is a diameter of the circle with center O and the tangent at a point P meets BA extended at T. if $\angle ABP = 40^{\circ}$, find $\angle PTA$.



ANS:

OB = OP (radii of same circle) In $\triangle BOP$, $\angle OBP = \angle OPB = 40^{\circ}$ (isoscele \triangle prop) ∠OPT = 90° (Angle between tangent and radius)

$$\angle$$
B + \angle BPT + \angle PTB = 180°
40° + 130° + \angle PTB = 180°
∴ \angle PTB = 10°

Q.33 Find the area of the segment AYB shown in Fig. 12.9, if radius of the circle is 21 cm

and AOB = 120°. (Use $\pi = \frac{22}{7}$).

Solution: Area of the segment AYB = Area of sector OAYB – Area of Δ OAB (1)

Now, area of the sector OAYB = $\frac{120}{360} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 462 \text{ cm}^2$

For finding the area of \triangle OAB, draw OM \perp AB as shown in Fig. 12.10. Note that OA = OB. Therefore, by RHS congruence, Δ AMO \cong Δ BMO.

So, M is the mid-point of AB and \angle AOM = \angle BOM = $\frac{1}{2} \times 120^{\circ} = 60^{\circ}$

So,

Also.

So, from Δ OMA,

 $AB = 2 AM = \frac{2 \times 21\sqrt{3}}{2} cm = 21\sqrt{3} cm$ $\frac{\mathrm{AM}}{\mathrm{OA}} = \sin 60^{\circ} = \frac{\sqrt{3}}{2}$ area of \triangle OAB = $\frac{1}{2}$ AB \times OM = $\frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$ cm²

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Therefore, area of the segment AYB = $\frac{21}{4}(88 - 21\sqrt{3}) \text{ cm}^{2}$ A bucket is 40 cm in 1:

Q.34 A bucket is 40 cm in diameter at the top and 28 cm in diameter at the bottom. Find the capacity of the bucket in litters, if it is 21 cm deep. Also, find the cost of tin sheet used in making the bucket, if the cost of tin is Rs. 1.50 per sq dm. Sol. Given: $r_1 = 20$

cm $r_2 = 14$ cm and h = 21 cm

Now, the required capacity (i.e. volume) of bucket = $\frac{\pi h}{3} (r_1^2 + r_1 r_2 + r_2^2) \cong$

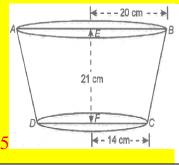
 $\frac{22 \times 21}{7 \times 3} (20^2 + 20 \times 14 + 14^2) \text{ cm}^3 = 22 \times 876 \text{ cm}^3 = 19272 \text{ cm}^3 = \frac{19272}{1000} \text{ liters} = 19.272 \text{ liters}.$

Now, I = $\sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(20 - 14)^2 + 21^2} cm = \sqrt{6^6 + 21^2} cm$ = $\sqrt{36 + 441} cm = \sqrt{477} cm \approx 21.84$

.. Total surface area of the bucket (which is open at the top)

 $= \pi \ell (r_1 + r_2) + \pi r_2^2 = \pi [(r_1 + r_2)\ell + r_2^2] = \frac{22}{7} [(20 + 14)21.84 + 14^2] = 2949.76 \text{ cm}^3$

.. Required cost of the tin sheet at the rate of Rs. 1.50 per dm² i.e., per 100 cm²



 $= Rs \frac{1.50 \times 2949.76}{100} \cong Rs 44.25$

"NOTHING IS TOO SMALL TO KNOW, AND
NOTHING IS TOO BIG TO ATTEMPT."

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