K.P.CLASSES

MATHEMATICS

CLASS-XII

TEST PAPER - 1

MAX. TIME = 3 HOURS **MAX. MARKS = 100**

GENERAL INSTRUCTIONS:-

- *All questions are compulsory.*
- *The question paper consists of 29 questions divided into three sections A, B and C.*
- *Section A contains 10 questions of 1 mark each, Section B is of 12 questions of 4 marks each and Section C is of 7 questions of 6 marks each.*
- *There is no overall choice. However, internal choice has been provided in four questions of four marks and two questions of six marks each.*
- *In question on construction, the drawing should be neat and exactly as per the given measurements.*
- *Use of calculators is not permitted. However you may ask for mathematical tables.*

SECTION-A

1. Let R be the relation in the set N given by R = {(a, b): a = $b-2, b>6$}. Choose the correct answer:
   a. $(2,4) \in R$.
   b. $(3,8) \in R$.
   c. $(6,8) \in R$.
   d. $(8,7) \in R$.
2. Let A= { 1, 2 ,3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.
3. If A and B are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix.
4. Find type of matrix is A which is both symmetric and skew-symmetric.
5. If |A| = -1 and |B| = 3, then what is the value of |3AB|.
6. Evaluate: $\int_{-1}^{1} |x| \ dx$.
7. What is the condition on the function f such that $\int_{-a}^{a} f(x) \ dx = 2 \int_{0}^{a} f(x) \, dx$?.
8. Evaluate: $\hat{\imath}.\hat{\jmath} + \hat{\jmath}.\hat{k} + \hat{k}.\hat{\jmath}$
9. Can a vector have direction angles $120^o, 45^0, 60^0$.
10. If α, β, γ *be the direction angles of a line then find the value of* $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

SECTION-B

11. Consider f : R→ R given by f(x) = 4x + 3. Show that f is invertible. Find the inverse of f.

    Or

    Determine which of the following binary operations on the set R are associative and which are commutative.

    (a). a * b = 1 $\forall\, a, b \in R$.

    (b). $a * b = \frac{a+b}{2} \ \forall\, a, b \in R$.
12. Solve: $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$.

    Or

    Simplify: $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x,\ -\frac{1}{\sqrt{2}} \le x \le 1$.
13. If a, b and c are real numbers, and $\begin{bmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{bmatrix} = 0$, Show that either a + b + c = 0 or a = b = c.

Or

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, show\ that\ A^2 - 5A + 7I = 0.$

14. Examine the continuity of f, where f is defined by $f(x) = \begin{cases} \sin x - \cos x\ , & if\ x \neq 0 \\ -1, & if\ x = 0 \end{cases}$.

15. Find $\frac{dy}{dx}, if\ (\cos x)^y = (\cos y)^x$.

Or

Verify Rolle's theorem for $f(x) = x^2 + 2x - 8\ on\ [-4, 2]$. Hence find 'c' if exist.

16. Find the equation of tangent to the curve $y = x^2 - 2x + 7$ which is
    a. Parallel to line $2x - y + 9 = 0$.
    b. Perpendicular to line $5x - 15y = 13$.

17. Integrate : $\int \frac{1}{x\ (x^n + 1)}\ dx$.

18. Solve: $\frac{dy}{dx} + (\sec x)y = \tan x\ ,\quad \left(0 \leq x \leq \frac{\pi}{2}\right)$.

19. Form the differential equation of the family of circles in the first quadrant which touch the coordinate axes.

20. Find the distance between the point P (6, 5, 9) and the plane determined by the points $A(3, -1, 2), B(5,2,4)$ and $C(-1, -1, 6)$.

21. Prove that: $\left[\vec{a} + \vec{b}\quad \vec{b} + \vec{c}\quad \vec{c} + \vec{a}\right] = 2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$.

22. Find the probability of throwing atmost 2 sixes in 6 throws of a single die.

SECTION-C

23. Obtain the inverse of the following matrix using elementary operations: $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

Or

Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

$x - y + 2z = 1;\ \ 2y - 3z = 1; 3x - 2y + 4z = 2.$

24. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 cm. Find the dimensions of the window to admit maximum light through the whole opening.

25. Evaluate $\int_1^4 (x^2 - x)\ dx$ Or

    Find the area bounded by curves $(x - 1)^2 + y^2 = 1\ and\ x^2 + y^2 = 1$.

26. Find the vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}\ \ and\ \ \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

27. A person wants to invest atmost Rs. 18000 in Savings Certificates and National Saving Bonds. According to rules, he has to invest atleast Rs. 4,000 in Savings Certificates and atleast Rs. 5000 in National Saving Bonds. Rate of interest on Savings Certificates is 9% per annum, that on National Saving Bonds is 11% per annum. Determine his investment in each scheme so as to earn maximum interest in a year.

28. There are three coins. One is two headed coin, another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows head. What is the probability that it was the two headed coin?

29. Evaluate: $\int_0^{\pi} \frac{x\ \tan\ x}{\sec\ x + \tan\ x}\ dx$.

Answers:-

1. C
2.
3.
4. A = 0 (null matrix).
5. -81
6. Ans = 1
7. $f(-x) = f(x)$.
8. 0
9. Yes
10. 2
11. $f^{-1}(y) = \frac{y-3}{4}$

    $or\ \ (a).\ both\ associative\ and\ commutative\ (b).\ commutative\ but\ not\ associative.$
12. $x = \frac{1}{6}$
13.
14. $f\ is\ continuous\ for\ all\ x\ \in R.$
15. $\frac{dy}{dx} = \frac{y\tan x + \log\cos y}{x\ tan y + \log\cos x}$. Or $c = -1 \in (-4, 2)$.
16. $(a).\ 2x - y + 3 = 0 \quad (b).\ 12x + 36y = 227.$
17. $\frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right) + C$
18. $y(\sec x + \tan x) = \sec x + \tan x - x + C$
19. $(x-y)^2(y_1^2+1) = (x + yy_1)^2$.
20. $Perpendicular\ distance = \frac{3\sqrt{34}}{17}\ units.$
21.
22. $\frac{35}{18}\left(\frac{5}{6}\right)^4$
23. $A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}\ or\ \ x = 0, y = 5, z = 3.$
24. $length = \frac{20}{\pi+4}, breadth = \frac{10}{\pi+4}.$
25. $I = \frac{27}{2}\ \ or\ \ Area = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}.$
26. $Equation\ of\ line:\ \vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$
27. Rs. 4000 in Saving certificates and Rs. 14000 in National Saving certificates.
28. 4/9
29. $I = \frac{\pi}{2}[\pi - 2]$.