PLEASURE TEST REVISION SERIES – 05

CLASS XII [2012 – 2013]

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Max.Marks: 100 **Time Allowed: 160 Minutes**

General Instructions:

*(a) The question paper consist of **29 questions** divided into **three sections A, B and C**. Section A comprises of **10 questions of one mark** each, section B comprises of **12 questions of four marks** each and section C comprises of **07 questions of six marks** each.*

*(b) All questions in Section A are to be answered in one word, one sentence or as per **the exact requirement** of the question.*

*(c) There is no overall choice. However, **internal choice** has been provided in **04 questions of four marks each** and **02 questions of six marks each**. You have to attempt only one of the alternatives in all such questions.*

(d) Use of calculators in not permitted. You may ask for logarithmic tables, if required.

SECTION – A

(Question numbers 01 to 10 carry one mark each.)

Q01. Write the principal value of $\text{cosec}^{-1}(-2)$.

Q02. Express as a single matrix: $4\begin{pmatrix}1 & 3\\ 1 & -4\end{pmatrix}-\frac{1}{2}\begin{pmatrix}6 & -4\\ -4 & 6\end{pmatrix}$.

Q03. If $f=\{(1,3),(2,5),(3,7)\}$ and $g=\{(3,-1),\ (5,8),(7,0)\}$. Find $gof(2)$.

Q04. Write the value of integral $\int \sin^2 x\,dx$.

Q05. Given a matrix A of order 3×3. Find |A . *adj* A|, if |A| = 9.

Q06. Evaluate: $\int_{-1}^{1}|x|\,dx$.

Q07. Find x, if $\begin{vmatrix}3 & 4\\ -5 & 2\end{vmatrix}=\begin{vmatrix}2x & 4\\ -5 & 3\end{vmatrix}$.

Q08. Find the value of λ, if $(2\hat{i}-\hat{j}+5\hat{k})\times(4\hat{i}+\lambda\hat{j}+10\hat{k})=\vec{0}$.

Q09. Write the value of p, such that the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{2p}$ and $\frac{3-x}{2}=\frac{y}{4}=\frac{z}{1}$ are perpendicular.

Q10. Find the projection of vector $3\hat{i}+4\hat{j}+5\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

SECTION – B

(Question numbers 11 to 22 carry four marks each.)

Q11. Discuss the continuity of the function $f(x)=\begin{cases}\dfrac{\cos x}{\frac{\pi}{2}-x},\ if\ x\neq\dfrac{\pi}{2}\\ 1,\ if\ x=\dfrac{\pi}{2}\end{cases}$ at $x=\frac{\pi}{2}$.

Q12. Prove that: $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$. **OR** Solve for x: $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4},\ x>0$.

Q13. Show that the equation of the perpendicular from the point (1, 6, 3) to the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ is $\frac{x-1}{0}=\frac{y-6}{-3}=\frac{z-3}{2}$ and the foot of perpendicular is (1, 3, 5) and the length of the perpendicular is $\sqrt{13}$ units.

Q14. Solve the differential equation: $(1-x^2)dy + xy\,dx = xy^2dx$.

OR Solve the differential equation: $(x^3+x^2+x+1)\frac{dy}{dx} = 2x^2+x$, given that $y=1$ when $x=0$.

Q15. Evaluate: $\int e^{2x}\left(\frac{\sin 4x - 2}{1-\cos 4x}\right)dx$.

Q16. Evaluate $\int_0^1 \frac{dx}{(x^2+a^2)(x^2+b^2)}$.

Q17. A bag contains 3 red coloured crackers and 4 white coloured crackers. Two crackers are selected one after the other without replacement. If the second selected cracker is given to be white, what is the probability that the first selected cracker is also white?
'Playing with crackers should be avoided.' Why?

Q18. Consider the function $f : R_+ \to (4,\infty)$ given by $f(x) = x^2+4$. Show that f is invertible, where R_+ represents the set of non-negative real numbers. Also find f^{-1}.

Q19. The volume of a cube is increasing at a constant rate. Prove that the rate of increase in surface area varies inversely as the length of edge of the cube.

OR Find the intervals in which the function $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is

(i) strictly increasing, and **(ii)** strictly decreasing.

Q20. Express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = 3\hat{i} + \hat{k}$ and the other is perpendicular to $\vec{b}$.

OR Find a unit vector perpendicular to plane ABC, where A, B, C are the points (3,–1, 2), (1,–1,–3) and (4,–3, 1) respectively.

Q21. If $y = \sin(m\sin^{-1}x)$, then show that: $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$.

Q22. If $p(x) = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$ then, prove that $p(x).p(y) = p(x+y)$.

SECTION – C

(Question numbers 23 to 29 carry six marks each.)

Q23. Using matrix method, solve the system of equations: $2x+6y=2,\ 3x-z=-8,\ 2x-y+z=-3$.

OR Using properties of determinants, prove that: $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$.

Q24. Use integrals to find area of the region enclosed between the curves $x^2+y^2=9$ and $(x-3)^2+y^2=9$.

Q25. Srishti is known to speak the truth 3 times out of 5 times. She throws a die and reports that it is '1'. What is the probability that it is actually a '1'? How does 'being a liar' impact our character development?

Q26. Find the area of the largest isosceles triangle having perimeter 18 metres.

Q27. Find the distance of the point (–2, 3,–4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$.

OR Find the coordinates of the reflection of the point (1, 2, 3) in the plane $x+2y+4z=38$.

Q28. Prove that: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$. Hence, evaluate: $\int_1^6 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}dx$.

Q29. A factory owner wants to purchase two types of machines, Machine A and Machine B for his factory. Machine A requires an area of 1000m^2 and 12 skilled men for running it and its daily output is 50 units, whereas Machine B requires 1200m^2 area and 8 skilled men, and its daily output is 40 units. If an area of 7600m^2 and 72 skilled men are available to operate the machines, set up L.P.P. to maximize the daily output and, hence solve it. "Use of machines has made the production of goods easier." Comment.