Question Paper
[CBSE ANNUAL EXAMINATION]

Max. Marks: 100 **Time Allowed: 3 Hours**

SECTION – A
(Question numbers 01 to 10 carry one mark each)

Q01. Find the value of $\int \frac{\log x}{x} dx$.

Q02. What is the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

Q03. If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then for what value of α is A an identity matrix?

Q04. What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes with *y*-axis?

Q05. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$.

Q06. What is the range of the function $f(x) = \frac{|x-1|}{(x-1)}$?

Q07. Find the minor of the element of second row and third column (a_{23}) in: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

Q08. Write the vector equation of the line: $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$.

Q09. What is the degree of the differential equation: $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$?

Q10. If $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$, then write the value of *k*.

SECTION – B
(Question numbers 11 to 22 carry four marks each)

Q11. Find all points of discontinuity of f, where f is defined as: $f(x) = \begin{cases} |x| + 3, & x \le -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \ge 3 \end{cases}$.

OR Find $\frac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{\frac{1}{x}}$.

Q12. Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x \in (0,1)$.

OR Prove that: $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$.

Q13. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Q14. Let $*$ be a binary operation on Q defined by $a * b = \frac{3ab}{5}$. Show that the operation $*$ is commutative as well as associative. Also find its identity element, if it exists.

Q15. Using elementary row operations, find inverse of matrix: $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$.

Q16. Evaluate: $\int_0^{\pi} \frac{x}{1+\sin x} dx$.

Q17. Find the Cartesian equation of the plane passing through the points $A(0,0,0)$ and $B(3,-1,2)$ and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$.

Q18. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-3\vec{b}\right)$ respectively, externally in the ratio $1:2$. Also, show that P is the midpoint of the line segment RQ.

Q19. Evaluate: $\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx$. **OR** Evaluate: $\int \frac{1-x^2}{x(1-2x)}dx$.

Q20. Find equations of the normals to the curve $y=x^3+2x+6$ which are parallel to the line: $x+14y+4=0$.

Q21. Find the particular solution of the differential equation satisfying the given conditions: $x^2dy+\left(xy+y^2\right)dx=0;\ y=1$ when $x=1$.

Q22. Find the general solution of the differential equation: $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$.

OR

Find the particular solution of the following differential equation satisfying the given conditions:

$$\frac{dy}{dx}=y\tan x\ ;\ \text{given that } y=1 \text{ when } x=0.$$

SECTION – C

(Question numbers 23 to 29 carry six marks each)

Q23. Using integration, find the area of the region: $\left\{(x,y):\frac{x^2}{9}+\frac{y^2}{4}\le 1\le\frac{x}{3}+\frac{y}{2}\right\}$.

OR Evaluate $\int_1^3\left(3x^2+2x\right)dx$ as the limit of sums.

Q24. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. Maximum number of hours available per day is 16. If the profit on a ring is ₹300 and that on a chain is ₹190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically.

Q25. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to both clubs. Find the probability of the lost card being of clubs.

Q26. Using properties of determinants, show that: $\begin{vmatrix}(b+c)^2 & ab & ca\\ ab & (a+c)^2 & bc\\ ca & bc & (a+b)^2\end{vmatrix}=2abc(a+b+c)^3$.

Q27. Find the value(s) of x for which $f(x)=\left[x(x-2)\right]^2$ is an increasing function. Also find the points on the curve, where the tangent is parallel to x-axis.

Q28. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

Q29. Write the vector equations of the lines given below and hence determine the distance between them: $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6};\ \frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}$.