TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

CODE:- AG-X-9999 पजियन क्रमांक **REG.NO:-TMC -D/79/89/36**

General Instructions :

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 5 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

सामान्य निर्देश :

1. सभी प्रश्न अनिवार्य हैं।
2. इस प्रश्न पत्र में 29 प्रश्न है, जो 3 खण्डों में अ, ब, व स है। खण्ड – अ में 10 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड – ब में 12 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको के हैं। खण्ड – स में 7 प्रश्न हैं और प्रत्येक प्रश्न 6 अंको का है।
3. प्रश्न संख्या 1 से 10 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
4. इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 4 प्रश्न 4 अंकों में और 2 प्रश्न 6 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
5. कैलकुलेटर का प्रयोग वर्जित हैं ।
6. कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ 5 हैं।
7. प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें।

Pre-Board Examination 2011 -12

Time : 3 Hours	अधिकतम समय : 3
Maximum Marks : 100	अधिकतम अंक : 100
Total No. Of Pages :5	कुल पृष्ठों की संख्या : 5

CLASS – XII CBSE MATHEMATICS

SECTION A

NOTE:- Choose the correct answer from the given four options in each of the Questions 1 to 3.

Q.1	If $\begin{pmatrix} x & y \\ x & y \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}=\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ then (x, y) is (A) (1, 1) (B) (1, –1) (C) (–1, 1) (D) (–1, –1) ans : c
Q.2	The area of the triangle with vertices (–2, 4), (2, k) and (5, 4) is 35 sq. units. The value of k is (A) 4 (B) – 2 (C) 6 (D) – 6 ans : d
Q.3	The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point (A) (1, 2) (B) (2, 1) (C) (1, – 2) (D) (–1, 2) ans : a
Q.4	Construct a 2 × 2 matrix whose elements aij are given by $a_{ij} = \begin{cases} \frac{\lvert -3i+j \rvert}{2} & if\ i \neq j \\ (i+j)^2 & if\ i = j \end{cases}$. ans : $\begin{pmatrix} 4 & 1/2 \\ 5/2 & 16 \end{pmatrix}$
Q.5	Find the value of derivative of $\tan^{-1}(e^x)$ w.r.t. x at the point $x = 0$. ans : 1/2
NOTE:-	Fill in the blanks in Questions 6 to 8.
Q.6	$\int \frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}\,dx =$ _____ ans : x +c
Q.7	If $a = 2\hat{i}+4\hat{j}-\hat{k}$ and $\hat{b}= 3\hat{i}-2\hat{j}+\lambda\hat{k}$ are perpendicular to each other, then λ=-----------ans : λ=-2
Q.8	The projection of $\vec{a} = \hat{i}+3\hat{j}+\hat{k}$ along $\vec{b} = 2\hat{i}-3\hat{j}+6\hat{k}$ is __________ ans :1/7

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Q.9 The 2 vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC, respectively of a ΔABC. Find the length of the median through A .ans : Median AD is given by

$$|\overrightarrow{AD}| = \frac{1}{2}\left|3\hat{i}+\hat{j}+5\hat{k}\right| = \frac{\sqrt{34}}{2}$$

Q.10 Evaluate: $\int_{-5}^{5}\left(\sin^{83} x + x^{123}\right)dx$. ans :0

SECTION B

Q.11 Solve the equation for x if $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}, x > 0$. Ans: $x^2 = \frac{3}{28}$ or $x = \frac{1}{2}\sqrt{\frac{3}{7}}$.

OR

Prove that : $\tan^{-1}\left[\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right] = \frac{x}{2}$.

Q.12 Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ where a, b, c are in A.P.

Q.13 Evaluate:

$\int_{0}^{1} x\left(\tan^{-1} x\right)^2 dx$.

Solution $I = \int_0^1 x(\tan^{-1} x)^2\, dx$.

Integrating by parts, we have

$$I = \frac{x^2}{2}\left[(\tan^{-1} x)^2\right]_0^1 - \frac{1}{2}\int_0^1 x^2 . 2\frac{\tan^{-1} x}{1+x^2}dx$$

$$= \frac{\pi^2}{32} - \int_0^1 \frac{x^2}{1+x^2} . \tan^{-1} x\, dx$$

$$= \frac{\pi^2}{32} - I_1 \text{, where } I_1 = \int_0^1 \frac{x^2}{1+x^2}\tan^{-1} x dx$$

Now

$$I_1 = \int_0^1 \frac{x^2+1-1}{1+x^2}\tan^{-1} x\, dx$$

$$= \int_0^1 \tan^{-1} x\, dx - \int_0^1 \frac{1}{1+x^2}\tan^{-1} x\, dx$$

Q.14 If $x = 2\cos\theta - \cos 2\theta$ & $y = 2\sin\theta - \sin 2\theta$ find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$. Ans :

$\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ is $\frac{3}{8}\sec^3\frac{3\pi}{4}\text{cosec}\frac{\pi}{4} = \frac{-3}{2}$

Q.15 If $x\sqrt{(1+y)} + y\sqrt{(1+x)} = 0$ then $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

Q.16 Water is dripping out at a steady rate of 1 cu cm/sec through a tiny hole at the vertex of the conical vessel, whose axis is vertical. When the slant height of water in the vessel is 4 cm, find the rate of decrease of slant height, where the semi vertical angle of the conical vessel is $\frac{\pi}{6}$.

Solution Given that $\frac{dv}{dt} = 1$ cm³/s, where v is the volume of water in the conical vessel.

From the Fig.6.2, $l = 4$cm, $h = l\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}l$ and $r = l\sin\frac{\pi}{6} = \frac{l}{2}$.

Therefore, $v = \frac{1}{3}\pi r^2 h = \frac{}{3}\frac{l^2}{4}\frac{\sqrt{3}}{2}l \quad \frac{\sqrt{3}}{24}l^3$.

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$$\frac{dv}{dt}=\frac{\sqrt{3}\,\pi}{8}l^2\frac{dl}{dt}$$

Therefore, $1=\frac{\sqrt{3}\pi}{8}16.\frac{dl}{dt}$

$$\Rightarrow \quad \frac{dl}{dt}=\frac{1}{2\sqrt{3}\pi}\text{ cm/s.}$$

Therefore, the rate of decrease of slant height = $\frac{1}{2\sqrt{3}\pi}$ cm/s.

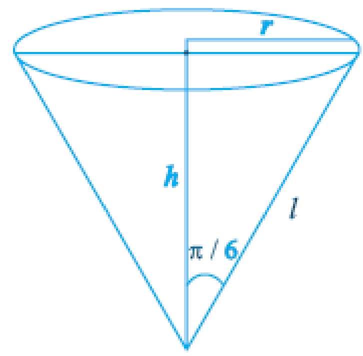

Fig. 6.2

OR

Find the intervals in which the function f given by $f(x)=x^3+\frac{1}{x^3}, x\neq 0$ is (i) increasing (ii) decreasing . ans :

$$f(x)=x^3+\frac{1}{x^3} \Rightarrow f'(x)=3x^2-\frac{3}{x^4} = \frac{3\left(x^6-1\right)}{x^4}=\frac{3\left(x^2-1\right)\left(x^4+x^2+1\right)}{x^4}$$

Thus f is increasing in $(-\infty,-1)\cup(1,\infty)$ Thus $f(x)$ is decreasing in $(-1,0)\cup(0,1)$

Q.17 Obtain a differential equation of the family of circles touching the x-axis at origin.

Ans: Equation of circle : $x^2+(y-a)^2=a^2$ Rrquired differential eqn $\left(x^2-y^2\right)y_1=2xy$

Q.18 Evaluate: $\int\frac{dx}{(\sin x+\sin 2x)}$ .sol: $I=\int\frac{dx}{\sin x(1+2\cos x)}=\int\frac{\sin x\,dx}{\sin^2 x(1+2\cos x)}$

$$=\int\frac{\sin x\,dx}{(1-\cos x)(1+\cos x)(1+2\cos x)}$$

Now differential coefficient of $\cos x$ is $-\sin x$ which is given in numerator and hence we make the substitution

$\cos x=t\Rightarrow-\sin x\,dx=dt$

$$\therefore\ I=-\int\frac{dt}{(1-t)(1+t)(1+2t)}$$

We split the integrand into partial fractions

$$\therefore\ I=-\int\left[\frac{1}{6(1-t)}-\frac{1}{2(1+t)}+\frac{4}{3(1+2t)}\right]dt \text{ etc.=}$$

$$\frac{1}{6}\log(1-\cos x)+\frac{1}{2}\log(1+\cos x)-\frac{2}{3}\log(1+2\cos x).$$

OR

Evaluate: $\int\cos 2\theta\log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)d\theta$. We know that

$$\log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)=\log\left(\frac{1+\tan\theta}{1-\tan\theta}\right)=\log\tan\left(\frac{\pi}{4}+\theta\right)$$

$$\int\sec\theta\,d\theta=\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$$

$$\therefore\ \int\sec 2\theta\,d\theta=\frac{1}{2}\log\tan\left(\frac{\pi}{4}+\theta\right)$$

$$\therefore\ 2\sec 2\theta=\frac{d}{d\theta}\log\tan\left(\frac{\pi}{4}+\theta\right) \quad \ldots..(i)$$

Integrating the given expression by parts, we get

$$I=\frac{1}{2}\sin 2\theta\log\tan\left(\frac{\pi}{4}+\theta\right)-\frac{1}{2}\int\sin 2\theta\,.\,2\sec 2\theta\,d\theta \quad \text{by (i)}$$

$$=\frac{1}{2}\sin 2\theta\log\tan\left(\frac{\pi}{4}+\theta\right)-\int\tan 2\theta\,d\theta = \frac{1}{2}\sin 2\theta\log\tan\left(\frac{\pi}{4}+\theta\right)-\frac{1}{2}\log\sec 2\theta.$$

Q.19 Solve the differential equation $:x\frac{d^2y}{dx^2}=1$ given that $y=1,\frac{dy}{dx}=0, when\ x=1.$ Ans.

$y=x\log x-x+2$

Q.20 Let $f(x)=x|x|$, for all $x\in R$. Discuss the derivability of $f(x)$ at $x=0$.

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Solution We may rewrite f as $f(x) = \begin{cases} x^2, \text{if } x \geq 0 \\ -x^2, \text{if } x < 0 \end{cases}$

Now $Lf'(0) = \lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^-} \frac{-h^2 - 0}{h} = \lim_{h \to 0^-} -h = 0$

Now $Rf'(0) = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{h^2 - 0}{h} = \lim_{h \to 0^-} h = 0$

Since the left hand derivative and right hand derivative both are equal, hence f is differentiable at $x = 0$.

Q.21 Find the shortest distance between the following lines : (x – 3)/1 = (y – 5)/–2 = (z – 7)/1 and (x + 1)/7 = (y + 1)/–6 = (z + 1)/1. Solution : S.D. = |{(4i + 6j + 8k).(– 4i – 6j – 8k)}/√116| = |(– 16 – 36 – 64)/√116| = |– 116/√116| = √116 = 2√29. [Ans.]

Q.22 A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and found to be hearts. Find the probability of the missing card to be a heart.

OR

Find the probability that in 10 throws of a fair die a score which is a multiple of 3 will be obtained in at least 8 of the throws. **Solution** Here success is a score which is a multiple of 3 i.e., 3 or 6. Therefore, p (3 or 6) = 2/ 6

Now P (at least 8 successes) = P (8) + P (9) + P (10)

$= {}^{10}C_8 \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right)^2 + {}^{10}C_9 \left(\frac{1}{3}\right)^9 \left(\frac{2}{3}\right)^1 + {}^{10}C_{10} \left(\frac{1}{3}\right)^{10}$

= 1 / 3 . $P(r) = {}^{10}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{10-r}$

$= \frac{1}{3^{10}} [45 \times 4 + 10 \times 2 + 1] = \frac{201}{3^{10}}$.

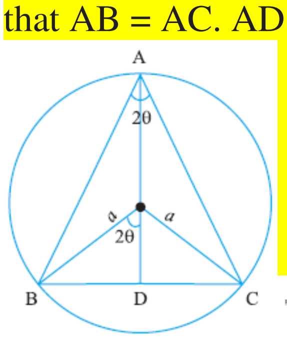
SECTION C

Q.23 Let the two matrices A and B be given by $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} and B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Verify that AB = BA = 6I, where I is the unit matrix of order 3 and hence solve the system of equations x – y = 3 ; 2x + 3y + 4z =17 ; y+ 2z =7 . ans : $AB = \begin{bmatrix} 6 & 0 & O \\ 0 & 6 & O \\ 0 & 0 & 6 \end{bmatrix}$ =6I There fore

$A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ & Hence $x = 2$, $y = 1$ and $z = 4$.;

Q.24 On the set **R**– {– 1}, a binary operation is defined by $a * b = a + b + ab$ for all $a, b \in$ **R** – {– 1}. Prove that * is commutative & associative property on **R** – {–1}. Find the identity element and prove that every element of **R** – {– 1}is invertible. Ans : Hence * is commutative on **R** – {–1}. **Identity Element :** Thus, 0 is the identity element for * defined on **R** – {–1}. **Inverse** : Hence, every element of **R** – {–1 }is invertible and the inverse of an element a is $-\frac{a}{a+1} \in R - \{-1\}$.

Q.25 An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of triangle is maximum when $\theta = \frac{\pi}{6}$. **Solution** Let ABC be an isosceles triangle inscribed in the circle with radius a such that AB = AC. AD = AO + OD = $a + a\cos 2\theta$ and BC = 2BD = $2a \sin 2\theta$.



Therefore, area of the triangle ABC i.e. $\Delta = \frac{1}{2} BC . AD$

$= \frac{1}{2} 2a \sin 2\theta . (a + a \cos 2\theta) = a^2 \sin 2\theta (1 + \cos 2\theta) \Rightarrow \Delta = a^2 \sin 2\theta + \frac{1}{2} a^2 \sin 4\theta$

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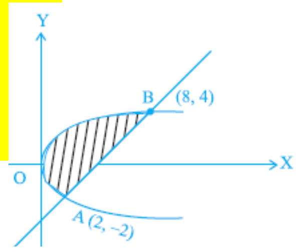
Therefore, $\frac{d\Delta}{d\theta} = 2a^2\cos 2\theta + 2a^2\cos 4\theta = 2a^2(\cos 2\theta + \cos 4\theta)$ $\frac{d\Delta}{d\theta} = 0 \Rightarrow \cos 2\theta = -\cos 4\theta = \cos(\pi - 4\theta)$

$\frac{d\Delta}{d\theta} = 0 \Rightarrow \cos 2\theta = -\cos 4\theta = \cos(\pi - 4\theta)$ Therefore, $2\theta = \pi - 4\theta \Rightarrow \theta = \frac{\pi}{6}$

$\frac{d^2\Delta}{d\theta^2} = 2a^2(-2\sin 2\theta - 4\sin 4\theta) < 0$ (at $\theta = \frac{\pi}{6}$). Therefore, Area of triangle is maximum when $\theta = \frac{\pi}{6}$.

Q.26 Find the area of the region bounded by the parabola $y^2 = 2x$ and the straight line $x - y = 4$. Ans : The intersecting points of the given curves are obtained by solving the equations $x - y = 4$ and $y^2 = 2x$ for x and y.

We have $y^2 = 8 + 2y$ i.e., $(y - 4)(y + 2) = 0$ which gives $y = 4, -2$ and $x = 8, 2$. Thus, the points of intersection are (8, 4), (2, –2). Hence

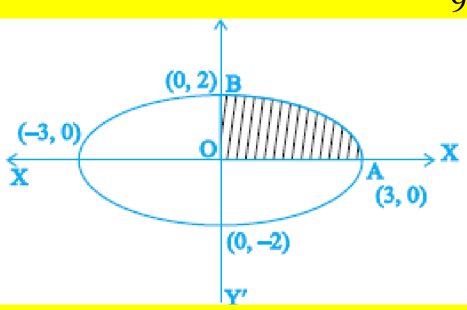


Area $= \int_{-2}^{4}\left(4 + y - \frac{1}{2}y^2\right)dy = \left|4y + \frac{y^2}{2} - \frac{1}{6}y^3\right|_{-2}^{4} = 18$ sq units.

OR

Find the area enclosed by the curve $x = 3\cos t$, $y = 2\sin t$ using integration . **Solution** Eliminating t as follows: $x = 3\cos t$, $y = 2\sin t$

$x = 3\cos t,\ y = 2\sin t \Rightarrow \frac{x}{3} = \cos t$, $\frac{x^2}{9} + \frac{y^2}{4} = 1$.which is the equation of an ellipse.



Required area $= 4\int_0^3 \frac{2}{3}\sqrt{9 - x^2}\,dx = 6\pi$ sq unit

Q.27 Find the co-ordinates of the foot of perpendicular from the point (2, 3, 7) to the plane $3x - y - z = 7$. Also, find the length of the perpendicular. Ans: Hence the co-ordinates of the foot of perpendicular is (5, 2, 6) and the length of perpendicular = $\sqrt{11}$.

OR

Find the equation of the plane containing the lines, $\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ .Find the distance of this plane from origin and also from the point (1, 1, 1) .ans : Hence equation of required plane is $-x + y + z = 0$ & Distance from (1, 1, 1) to the plane is $\frac{1}{\sqrt{3}}$.

Q.28 Two cards are drawn successively without replacement from well shuffled pack of 52 cards. Find the probability distribution of the number of kings. Also, calculate the mean and variance of the distribution. ANS : Let x denote the number of kings in a draw of two cards. Note that x is a random variable which can take the values 0, 1, 2.

Now $P(x=0) = P(\text{no king}) = \frac{^{48}C_2}{^{52}C_2} = \frac{\frac{48!}{2!(48-2)!}}{\frac{52!}{2!(52-2)!}} = \frac{48\times 47}{52\times 51}\ \frac{188}{221}$

P (x = 1) = P (one king and one non-king) = $\frac{^4C_1\ 48C_1}{^{52}C_2} = \frac{32}{221}$ Thus, the probability distribution of x is

and $P(x=2) = P(\text{two kings}) = \frac{^4C_2}{^{52}C_2} = \frac{4\times 3}{52\times 51} = \frac{1}{221}$ Thus, the probability distribution of x is

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x	0	1	2
P x	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

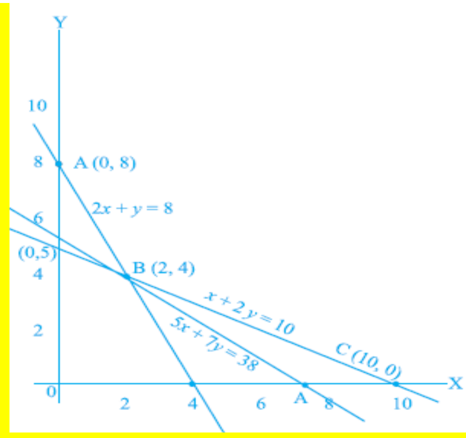
Now mean of $x = E(x) = \sum_{i=1}^{n} x_i P(x_i)$

$$= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + \frac{2 \times 1}{221} = \frac{34}{221}$$

VAR (x)= $E(x^2) - [E(x)]^2 = \left(\frac{36}{221}\right) - \left(\frac{34}{221}\right)^2 = \frac{6800}{221^2}$ Therefore standard deviation $\sqrt{\text{var}(x)} = \frac{\sqrt{6800}}{221}$ 0.37

Q.29 A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contains at least 8 units of Vitamin A and 10 units of Vitamin C. Food 'I' contains 2 units/kg of Vitamin A and 1 unit/kg of Vitamin C. Food 'II' contains 1 nit/kg of Vitamin A and 2 units/kg of Vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem to minimize the cost of such a mixture and solve it graphically.

ANS : Let the mixture contains x kg of food I and y kg of food II. Thus we have to minimize $Z = 50x + 70y$



Subject to : $2x + y \geq 8$; $x + 2y \geq 10$; $x, y \geq 0$. The feasible region

determined by the above inequalities is an unbounded region. Vertices of feasible region are A (0, 8) B (2, 4) ; C(10, 0) .Now value of Z at A (0, 8) = 50·0+70·8=560. B(2, 4) = 380 ; C(10, 0) = 500 . Hence, the optimal mixing strategy for the dietician would be to mix 2 kg of food I and 4 kg of food II to get the minimum cost of the mixture i.e Rs 380.

___________x__________

A MAN WHO DOESN'T TRUST HIMSELF ; CAN NEVER TRULY TRUST ANYONE ELSE

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