PLEASURE TEST REVISION SERIES – 08	SUBJECT – MATHEMATICS
By: OP GUPTA (+91- 9650 350 480)	FOR CLASS XII

Max. Marks: 100 **Time Allowed: 180 Minutes**

[SECTION – A]

Q01. Let * be a binary operation defined by a*b = 2a + b – 3, find the value of 3*4.

Q02. Write the value of: $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.

Q03. What is the value of Δ, if $\Delta = \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$?

Q04. Check if the function $-\frac{x^3}{3}+x^2-x+\frac{3}{2}$ is decreasing in R.

Q05. For what value of 'm' and 'p', is the matrix $\begin{pmatrix} 0 & 5 & -3 \\ -5 & m & 4 \\ p & -4 & 0 \end{pmatrix}$ skew-symmetric?

Q06. Show that a powerful bomb shot along the line of fire $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ will never hit a helicopter flying in the plane 2x + 4y – 4z + 11 = 0.

Q07. Write the number of binary operations that can be defined on the set {1, 2}.

Q08. Let $\vec{a}$ and $\vec{b}$ are non-collinear vectors. For what value of x, the vectors $\vec{c}=(x-2)\vec{a}+\vec{b}$ and $\vec{d}=(2x+1)\vec{a}-\vec{b}$ are collinear?

Q09. Write a unit vector perpendicular to the vectors $\vec{a}$ and $\vec{b}$ both, if it is given that $\vec{a}=3\hat{i}+2\hat{j}+6\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+2\hat{k}$.

Q10. Evaluate: $\int \frac{3\cos x}{2\sin^2 x}dx$.

[SECTION – B]

Q11. Discuss the differentiability of $f(x)=\begin{cases} 1-x, \text{ if } x<1 \\ (1-x)(2-x), \text{ if } 1\le x\le 2 \\ 2-x, \text{ if } x>2 \end{cases}$ at x = 2.

(OR) A car driver is driving a car on the dangerous path given by

$$f(x)=\begin{cases} \frac{1-x^m}{1-x}, \text{ if } x\ne 1 \\ m-1, \text{ if } x=1 \end{cases}, m \in N.$$

Find the dangerous point (point of discontinuity) on the path. Whether the driver should pass that point or not? Justify your answer.

Q12. Let $\vec{a}=2\hat{i}+\hat{k}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=4\hat{i}-3\hat{j}+7\hat{k}$ be three vectors. Determine a vector $\vec{r}$ which satisfies the condition $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$.

Q13. Express $\cos^{-1}\sqrt{\frac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}$ in simplest form.

(OR) Solve: $\sec^2\tan^{-1}2+\text{cosec}^2\cot^{-1}3=x$.

Q14. Evaluate: $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$.

Q15. Solve the differential equation: $(x-\tan^{-1}y)dy+(y^2+1)dx=0$.

Q16. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

(OR) Water is dripping out from a conical funnel at a uniform rate of $4cm^3/s$ through a tiny hole at the vertex in the bottom. When the slant height of the water is 3cm, find the rate of decrease of the slant height of the water-cone. Given that the vertical angle of funnel is 120°.

Q17. A chairman is biased so that he selects his relatives for a job 3 times as likely as others. If there are 3 posts for a job, find the probability distribution for selection of persons other than his relatives.
If the chairman is biased then, which value of life will be demolished?

Q18. Find the distance of the point (– 2, 4, – 5) from the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

Q19. Evaluate: $\int \frac{dx}{\sin(x-\alpha)\sin(x-\beta)}$. **(OR)** Evaluate: $\int \frac{(x+x^3)^{1/3}}{x^4}dx$.

Q20. Let f, g : R→R be defined as f (x) = |x| and g (x) = [x], where [x] denotes greatest integer less than or equal to x. Evaluate: $\frac{(gof)\left(-\frac{5}{3}\right)-(fog)\left(-\frac{5}{3}\right)}{(fo(gof))\left(-\frac{5}{3}\right)}$.

Q21. If $y^2=4ax$, then evaluate: $\left(\frac{d^2y}{dx^2}\right).\left(\frac{d^2x}{dy^2}\right)$.

Q22. Using properties of determinants, prove that: $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}=(a^3+b^3+c^3-3abc)^2$.

[SECTION – C]

Q23. Find the area bounded by $x^2+y^2=25,\ 4y=\left|4-x^2\right|,\ x=0,\ y=0$ which is lying above x-axis.

Q24. Three friends A, B and C visited a Super Market for purchasing fresh fruits. A purchased 1kg apples, 3kg grapes and 4kg oranges and paid ₹800. B purchased 2kg apples, 1kg grapes and 2kg oranges and paid ₹500. While C paid ₹700 for 5kg apples, 1kg grapes and 1kg oranges. Find the cost of each fruit per kg by using matrix method. Why are the fruits good for health?

(OR) Using elementary operations, find the inverse of $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, if it exists.

Q25. A manufacturer has three machine operators A (skilled), B (semi-skilled) and C (non-skilled). The first operator A produces 1% defective items whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of time, B in the job for 30% of the time and C is on the job for 20 % of the time. A defective item is produced, what is the probability that it was produced by B? What is the value of skill in industries?

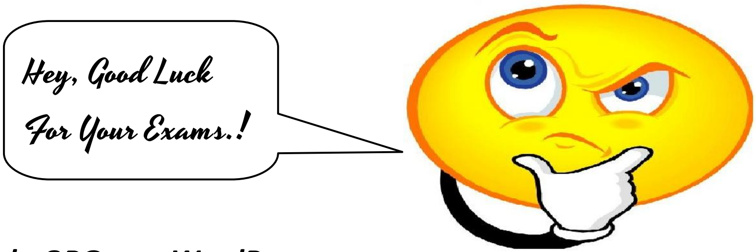
Q26. A bird at A(7, 14, 5) in space wants to reach a point P on the plane 2x + 4y – z = 2 when AP is least. Find the position of P and also the distance AP travelled by the bird.

Q27. Evaluate $\int_0^{\pi/2} \log \operatorname{cosec} x\, dx$, by using properties of the definite integral.

Q28. Avinash has been given two lists of problems from his mathematics teacher with the instructions to submit not more than 100 of them correctly solved for marks. The problems in the first list are worth 10 marks each and those in the second list are worth 5 marks each. He knows from past experience that he requires on an average of 4 minutes to solve a problem of 10 marks and 2 minutes to solve a problem of 5 marks. He has other subjects to worry about; he cannot devote more than 4 hours to his mathematics assignment. With reference to manage his time in best possible way, how many problems from each list shall he do to maximize his marks? What is the importance of time management for the students?

Q29. If PA and QB be two vertical poles of height 16m and 22m at points A and B respectively such that AB= 20m then, find the distance of a point R on AB from the point A such that RP^2+RQ^2 is minimum.

(OR) A point P is given on the circumference of a circle of radius r. The chord QR is parallel to the tangent line at P. Find the maximum area of the triangle PQR.



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ANSWERS

Q01. 7

Q02. $\frac{2\pi}{5}$

Q03. $a^2+b^2+c^2+d^2$

Q04. Decreasing function

Q05. 0, 3

Q06. Show that the line is parallel to the plane *i.e.,* the line is at right angle the normal vector of the plane.

Q07. $2^{2\times 2}=16$

Q08. 1/3

Q09. $\frac{4\hat{k}-8\hat{i}}{4\sqrt{5}}$

Q10. $-\frac{3}{2}\operatorname{cosec} x+k$

Q11.OR Point of discontinuity: x = 1. No, because life is precious so vehicles should be driven carefully.

Q12. $2\hat{k}-8\hat{j}-\hat{i}$

Q13. $\frac{1}{2}\tan^{-1}x$ OR $x=15$

Q14. $\left(\frac{\pi}{2}\right)^2$

Q15. $x=\tan^{-1}y+k\,e^{-\tan^{-1}y}-1$

Q16. 9.72cm^3 OR $\frac{32}{27\pi}$ cm

Q17. **Probability Distribution:**

X	0	1	2	3
P(X)	27/64	27/64	9/64	1/64

Values lost by chairman:

Honesty, integrity.

Q18. $\sqrt{\frac{37}{10}}\,units$

Q19. $\frac{1}{\sin(\alpha-\beta)}\log\left|\frac{\sin(x-\alpha)}{\sin(x-\beta)}\right|+k$ OR $-\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{4/3}+k$

Q20. –1

Q21. $-2a/y^3$

Q23. $\frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right)+2$ sq.units

Q24. Let cost of each fruit be x, y, z respectively. Then solve the equations so formed by using matrix method. So, x = 100, y = 100, z = 100. Hence the cost of each fruit is ₹100 per kg. **Importance of fruits:** Fruits contain nutrients and vitamins which help our body in its proper growth and maintenance.

OR $\begin{bmatrix} 1/3 & 1/3 & -2/3 \\ -2/3 & 1/3 & -2/3 \\ 1/3 & -1/6 & 5/6 \end{bmatrix}$

Q25. 15/34

Q26. P(1, 2, 8), AP = $\sqrt{189}$ *units*

Q27. $\frac{\pi}{2}\log 2$

Q28. 20 problems from first list and 80 problems from second list. Students who divide the time for each subject per day according to their need don't feel burden of any subject before the examination.

Q29. $10m$ OR $\frac{3\sqrt{3}\,r^2}{4}$ *sq.units* .

I (*i.e.* OP Gupta) have been training the students of XI, XII & Engineering aspirants since year 2006. I conduct my classes at Najafgarh, New Delhi. Being a die-hard lover of Mathematics, I've completely devoted myself for this science!

With lots of love and affections,
Wish you all the very best for your examinations & the beautiful life ahead!