PLEASURE TEST REVISION SERIES – 07
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SUBJECT – MATHEMATICS FOR CLASS XII

Max. Marks: 100 **Time Allowed: 180 Minutes**

(SECTION – A)

Q01. Write the value of $\int \frac{1}{1+e^x}dx$.

Q02. Solve: $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x;\ x>0$.

Q03. For what value of x, is the matrix $\begin{pmatrix} 3-2x & x+1 \\ 2 & 4 \end{pmatrix}$ a singular matrix?

Q04. If $f(x)=2x+5$ and $g(x)=x^2+1$ be two real valued functions, find fog.

Q05. Write the number of possible matrices of order 3×3 with each entry being either 0 or 1.

Q06. For any vector $\vec{r}$, evaluate: $\left(\vec{r}.\hat{\mathrm{i}}\right)\hat{\mathrm{i}}+\left(\vec{r}.\hat{\mathrm{j}}\right)\hat{\mathrm{j}}+\left(\vec{r}.\hat{\mathrm{k}}\right)\hat{\mathrm{k}}$.

Q07. Find f^{-1}, if $f(x)=\frac{3x-2}{5}$.

Q08. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $|\vec{a}+\vec{b}|=1$, then find $|\vec{a}-\vec{b}|$.

Q09. Write a vector parallel to $-\frac{\vec{a}}{|\vec{a}|}$ if it is given that $\vec{a}=3\hat{\mathrm{i}}-2\hat{\mathrm{j}}+6\hat{\mathrm{k}}$.

Q10. Evaluate: $\int_0^{\pi/2}\sin^2 x\,dx$.

(SECTION – B)

Q11. Discuss the continuity of the function $f(x)=\begin{cases}(\sin 3x)/(\tan 2x), & if\ x<0\\ 3/2, & if\ x=0\\ \frac{\log(1+3x)}{e^{2x}-1}, & if\ x>0\end{cases}$ at $x=0$.

Q12. Evaluate: $\int\sqrt{\frac{1-3x}{1+3x}}dx$.

(OR) Evaluate: $\int x\sqrt{\frac{64-x^2}{64+x^2}}dx$.

Q13. A vector $\vec{\alpha}$ of magnitude 8 units is inclined to *x*-axis at 45°, *y*-axis at 60° and an acute angle with the *z*-axis. If a plane passes through a point ($\sqrt{2}$,–1, 1) and is normal to $\vec{\alpha}$, find its equation in the vector form.

Q14. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, $-\frac{\pi}{2}<x<\frac{3\pi}{2}$ in the simplest form.

(OR) Prove that: $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)=\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$.

Q15. Evaluate: $\int\left(\log\log x+\frac{1}{(\log x)^2}\right)dx$. Discuss the importance of integration (*unity*) in life.

Q16. **a)** Write the order and degree (*if defined*) of: $y+\frac{dy}{dx}=\int y\,dx$.

b) Solve the differential equation: $\left(3xy+y^2\right)dx+\left(x^2+xy\right)dy=0$.

Q17. Show that the curves $y=ae^x$ and $y=be^{-x}$ cut at the right angles if $ab=1$.

(OR) Find the intervals in which the function $f(x)=\frac{x}{\log x}$ is increasing and/or decreasing.

Q18. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3, |\vec{b}|=4, |\vec{c}|=5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a}+\vec{b}+\vec{c}|$.

Q19. Show that the relation R in the set R of real numbers, defined as $\mathrm{R}=\{(a,b):a\le b^2\}$ is neither reflexive nor symmetric nor transitive.

Q20. A clever student used a biased coin so that the head is 3 times as likely to occur as tail. If the coin is tossed twice find the probability distribution and mean of numbers of tails. Is this a good tendency?

Q21. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a^3(x^3-y^3)$, then show that: $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$.

(OR) If $\sqrt{1+x^2}+\sqrt{1+y^2}=a\left(x-y\right)$, then show that: $\frac{dy}{dx}=\sqrt{\frac{1+y^2}{1+x^2}}$.

Q22. Using properties of determinants, prove that: $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}=(ab+bc+ca)^3$.

[SECTION – C]

Q23. If a triangular field is bounded by the lines x + 2y = 2, y – x = 1 and 2x + y = 7. Use integration to compute the area of the field. Hence find the followings:
(i) If in each square unit area, 4 trees may be planted. Find the number of trees that can be planted in the field. (ii) Why plantation of trees is necessary?

Q24. An amount of ₹600 crores is spent by the government in three schemes. Scheme A is for saving girl child from those parents who don't want girl child and get the abortion before her birth. Scheme B is for saving of newlywed girls from death due to dowry. Scheme C is planning for good health for senior citizen. Now twice the amount spent on Scheme C together with amount spent on Scheme A is ₹700 crores. And three times the amount spent on Scheme A together with amount spent on Scheme B and Scheme C is ₹1200 crores. Find the amount spent on each scheme using matrices? What is the importance of saving girl child from the abortion before her birth?

(OR) Given the matrices $\mathrm{A}=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $\mathrm{B}=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Verify that $\mathrm{AB}=\mathrm{BA}=6\mathrm{I}$.

Hence solve the system of equations: $x-y=3,\ 2x+3y+4z=17,\ y+2z=7$.

Q25. In a group of students, 200 attend coaching classes, 400 students attend school regularly and 600 students study themselves with help of peers. The probability that a student will succeed in life who attend coaching classes, attend school regularly and study themselves with help of peers are 0.1, 0.2 and 0.5 respectively. One student is selected who succeeded in life, what is the probability that he study himself with help of peers. What type of study can be considered for the success in life and why?

Q26. Find the length and the equation of the line of shortest distance between the lines:
$\frac{x-3}{3}=\frac{8-y}{1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. Also find the points where the line of shortest distance meets the given lines.

Q27. Evaluate: $\int_0^{3/2}\left|x\cos\pi x\right|dx$.

Q28. If a class XII student aged 17 years, rides his motor cycle at 40km/hr, the petrol cost is ₹2 per km. If he rides at a speed of 70km/hr, the petrol cost increases ₹7per km. He has ₹100 to spend on petrol and wishes to cover the maximum distance within one hour. Express this as an L.P.P. and solve graphically. (i) What is benefit of driving at an economical speed? (ii) Should a child below 18 years be allowed to drive a motorcycle? Give reasons.

Q29. Show that height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height *h* and semi vertical angle 60° is one-third that of the cone and the greatest volume of cylinder is given as: $\frac{4}{9}\pi h^3$.

(OR) A square tank of capacity 250 cubic meters has to be dug out. The cost of the land is ₹50 per square meter. The cost of digging increases with the depth and for the whole tank, it is ₹$(400\times h^2)$, where *h* meters is the depth of the tank. What should be the dimensions of the tank so that the cost is minimum?

ANSWERS

Q01. $x - \log|1 + e^x| + k$ Q02. $\frac{1}{\sqrt{3}}$ Q03. 1 Q04. $2x^2 + 7$

Q05. 2^9 Q06. $\vec{r}$ Q07. $f^{-1}(x) = \frac{5x+2}{3}$ Q08. $\sqrt{3}$

Q09. $-\frac{1}{7}(3\hat{i} - 2\hat{j} + 6\hat{k})$ Q10. $\frac{\pi}{4}$ Q11. Continuous

Q12. $\frac{1}{3}\sin^{-1} 3x + \frac{\sqrt{1-9x^2}}{3} + k$ OR $32\sin^{-1}\left(\frac{x^2}{64}\right) + \frac{1}{2}\sqrt{4096 - x^4} + k$

Q13. $\vec{r}.(\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}) = 2$ Q14. Let $y = \tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right) \Rightarrow y = \begin{cases} \frac{\pi}{4} + \frac{x}{2}, \text{ if } -\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2} \\ -\frac{\pi}{4} - \frac{x}{2}, \text{ if } \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \end{cases}$

Q15. $x \log\log x - \frac{x}{\log x} + k$ Q16. (a) Order: 2, Degree: 1 (b) $k = x^2\,(y^2 + 2\,x\,y)$

Q17. Increasing in (e, ∞) and, decreasing in $(*0, e) - 1$ Q18. $5\sqrt{2}$

Q20. **Probability Distribution:**

X	0	1	2
P(X)	9/16	6/16	1/16

Mean = 1/2.

1. No, it may be good once or twice but not forever.

2. Honesty pays in a long run.

Q23. 6sq.units (i) 24 tress (ii) Plants provide us oxygen and play major role in rain, so plantation is essential for all living beings.

Q24. ₹300crores, ₹200crores and ₹100crores. (i) In our country, male population is more than female population. (ii) It is essential for a human being to save the life of all. OR $x = 2, y = -1, z = 4$

Q25. 0.75. 'Self studies with the help of peers' is best as through it students can get the knowledge in depth of each concept. But students should be regular in school and if they feel need they could join different classes.

Q26. Eq. of line of Shortest Distance : $\frac{x-3}{-2} = \frac{y-8}{-5} = \frac{z-3}{1}$,

Length: $3\sqrt{30}$ *units*, Req.Points: $(3,8,3), (-3,-7,6)$. Q27. $\frac{5}{2\pi} - \frac{1}{\pi^2}$

Q28. Max. $Z = x + y$. Subject to constraints: $x/40 + y/70 \le 1$, $2x + 7y \le 100$; $x, y \ge 0$. Here x & y represents the distance travelled by the boy at speed of 40km/hr & 70km/h respectively.
So x = 1560/41km, y = 140/41km. (i) It saves petrol. It saves money. (ii) No, because according to the law driving license is issued when a person is above the 18 years of age.

Q29. Let the length and breadth of square tank be x metres. So, Cost Function, $C = 50\,x.x + 400\,h^2 \Rightarrow C = 50(V/h) + 400h^2$ [Using $V = x.x.h \Rightarrow x^2 = V/h$]. Hence the dimensions of the square tank are given as: length = breadth = 10m and depth = h = 5/2m.

With lots of love and blessings,
Wish you all the very best for your examinations and beautiful life ahead!