PLEASURE TEST REVISION SERIES – 06

MATHEMATICS

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Time : 03 Hrs. **Max. Marks :100**

SECTION – A

01. Show that the binary operation $*$ defined by $a*b = ab + 1$ on Q is commutative.

02. Find a matrix X such that $B - 2A + X = O$, where $A = \begin{bmatrix} 5 & 3 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 \\ 3 & 1 \end{bmatrix}$.

03. The income I of Dr.Rastogi is given by $I(x) = ₹\ (x^3 - 3x^2 + 5x)$. Can an insurance agent ensure him for the growth of his income?

04. Solve : $\tan^{-1}2x + \tan^{-1}3x = \pi/4$.

05. Write the values of $x - y + z$ from the following equation: $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$.

06. Evaluate : $\int \frac{dx}{x\cos^2(1+\log x)}$.

07. Evaluate : $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos^4 x\, dx$.

08. If $|\vec{a}| = 5$; $|\vec{b}| = 13$ and $|\vec{a} \times \vec{b}| = 25$, find $\vec{a} \cdot \vec{b}$.

09. The Cartesian equation of a line AB is $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$. Find the direction cosines of a line parallel to AB.

10. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$; $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

SECTION – B

11. If $f : R - \left\{\frac{7}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ be defined as $f(x) = \frac{3x+4}{5x-7}$ and $g : R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{7}{5}\right\}$ be defined as $g(x) = \frac{7x+4}{5x-3}$. Show that $gof = I_A$ and $fog = I_B$ where $B = R - \left\{\frac{3}{5}\right\}$ and $A = R - \left\{\frac{7}{5}\right\}$.

12. Use properties of determinants to prove : $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$.

OR Using properties of determinants, prove that : $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$.

13. Solve the following differential equation : $(1+x^2)\frac{dy}{dx} + y = \tan^{-1}x$.

14. For what value of k is the following function f (x) continuous at x = 2?

$$f(x) = \begin{cases} 2x+1; & x < 2 \\ k; & x = 2 \\ 3x-1; & x > 2 \end{cases}.$$

15. A survey revealed that 70% men and 30% women eat pan-masala. 10% of these men and 20% of these women eat brand X pan-masala. What is the probability that a person seen eating brand X will be a man? Why would you discourage intake of pan-masala?

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16. Find the intervals in which the function $f(x) = \sin x - \cos x;\ 0 \le x \le 2\pi$ is
(i) increasing and/or (ii) decreasing.

17. Prove that : $2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

OR Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.

18. Prove that : $\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\cos^4 x + \sin^4 x}dx = \frac{\pi^2}{16}$.

19. Find the particular solution of differential equation : $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$, if $y(1) = 0$.

20. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$, $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$ and $\vec{a} \ne 0$, then prove that $\vec{b} = \vec{c}$.

21. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.

OR If $y = [\log(x + \sqrt{1+x^2})]^2$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 2 = 0$.

22. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+1}{2}$.

SECTION – C

23. Given that $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ then, find the product AB. Hence using this product solve the system of equations : $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

24. A milkman is having a vessel in the shape of a right circular cylinder, which is open at the top and has a given surface area. Show that the vessel will acquire the maximum amount of milk if its height is equal to the radius of its base. "Intake of milk proves good for health." How?

25. Evaluate : $\int[\sqrt{\tan x} + \sqrt{\cot x}]dx$.

26. Using the method of integration, find the area of the region bounded by the following lines :
$2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

OR Make a rough sketch of the region given below and find the area using the method of integration : $\{(x,y): 0 \le y \le x^2 + 3, 0 \le y \le 2x + 3, 0 \le x \le 3\}$.

27. In a bolt factory, machines A, B and C, manufacture respectively 25%, 35%, 40% of the total bolts. Of their output 5%, 4% and 2% respectively are defective bolts. A bolt is drawn at the random and is found to be defective. Find the probability that it is manufactured by machine B. 'Machines have proved beneficial for mankind.' Comment.

28. An aeroplane can carry a maximum of 200 passengers. A profit of ₹400 is made on each first class ticket and a profit of ₹300 is made on each economy class ticket. The airline reserves at least 20 seats for first class. However, at least 4 times as many passengers prefer to travel by economy class to by the first class. Determine how many of each type ticket must be sold in order to maximize the profit for the airline. What is the maximum profit? Frame an L.P.P and solve it graphically.

29. Find the image Q of the point P(1, 2, 3) in the plane $x + 2y + 4z = 38$.
Also find the perpendicular distance from the point to the plane. Hence write the vector equation of line PQ.

OR A line makes angles α, β, γ and δ with the diagonals of a cube, prove that :

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = 8/3.$$

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ANSWERS

Q02. $\begin{bmatrix} 10 & 8 \\ -9 & 1 \end{bmatrix}$

Q03. Yes. Show that $I'(x) > 0$ for all $x \in R$.

Q04. 1/6

Q05. 1

Q06. tan (1 + log x) + k

Q07. Show that $\sin^5 x \cos^4 x$ is an odd function. So, using the property: $\int_{-a}^{a} f(x)dx = 0$ if $f(x)$ is an odd function, we get: $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos^4 x \, dx = 0$

Q08. 60

Q09. Parallel lines have same set of d.c.'s. So, d.c.'s of line parallel to AB are: $\sqrt{\frac{3}{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

Q10. $\frac{1}{\sqrt{22}}(3\hat{i} - 3\hat{j} + 2\hat{k})$

Q11. NCERT Part I Example 17, Page 13

Q13. $y = \tan^{-1} x + k e^{-\tan^{-1} x} - 1$

Q14. k = 5

Q15. 49/67

Q16. (i) $[0, 3\pi/4] \cup [7\pi/4, 2\pi]$ (ii) $[3\pi/4, 7\pi/4]$

Q17. x = 0

Q19. log x = 1 – cos (y/x)

Q21. –3/2

Q22. $\frac{10}{\sqrt{59}} units$

Q23. AB = 8I, x = 3, y = –2, z = –1

Q25. NCERT Part II Example 41, Page 350: $\sqrt{2}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + k$

Q26. 7/2 sq.units OR 50/3sq.units

Q27. 196/449

Q28. ₹64000

Q29. Q (3, 6, 11), $\sqrt{21} units$. Eq. of PQ: $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + 6\hat{j} + 11\hat{k})$.

With lots of love & blessings,

All the very best for your examinations and the beautiful life ahead!

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