SAMPLE TEST PAPER – 04

Max. Marks: 100 **Time Allowed: 3 Hours**

SECTION – A

Q01. Check whether the relation R defined in the set $A=\{1,2,3\}$ as $R=\{(a,b):b=a+1\}$ is reflexive, symmetric or transitive.

Q02. Find the value of k for which the matrix $\begin{pmatrix} k & 2 \\ 3 & 4 \end{pmatrix}$ has no inverse.

Q03. Evaluate: $\int\left[1+2\tan x(\tan x+\sec x)\right]^{1/2}dx$.

Q04. Find the value of x if the area of triangle is 35sq.units with the vertices as $(x, 4)$, $(2,-6)$, $(5, 4)$.

Q05. Write down a unit vector in XY–plane, making an angle of thirty degrees with the positive direction of X–axis.

Q06. Evaluate: $\int\frac{\cos 2x-\cos 2a}{\cos x-\cos a}dx$.

Q07. Write one of the range of $\sec^{-1}x$ other than its principal branch.

Q08. Find the perpendicular distance of (2, 5, 6) from XY–plane.

Q09. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, find the value of x for which the vectors $\vec{l}=(2x+1)\vec{a}-\vec{b}$ and $\vec{m}=(x-2)\vec{a}+\vec{b}$ are collinear.

Q10. If $\vec{a}=\vec{b}+\vec{c}$ then, find $[\vec{a}\ \ \vec{b}\ \ \vec{c}]$, if $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=\hat{j}-\hat{i}+\hat{k}$.

SECTION – B

Q11. A girl walks 4km towards west, then she walks 3km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

OR If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+\hat{j}+2\hat{k}$, find a unit vector which is a linear combination of $\vec{a}$ and $\vec{b}$ and is also perpendicular to $\vec{a}$.

Q12. If $y=\sin(m\sin^{-1}x)$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$.

OR If $f(x)=\log\cos x$ then, find $f'(x)$ by using definition of derivatives.

Q13. Find all the local maximum and minimum values of the function $f(x)=\sin 2x-x,\ -\frac{\pi}{2}<x<\frac{\pi}{2}$.

Q14. If $x^y=e^{x-y}$, prove that $\frac{dy}{dx}=\frac{\log x}{(\log ex)^2}$.

Q15. Solve: $e^x\tan y\,dx+\sec^2 y(1-e^x)dy=0$. **OR** Solve: $(3xy+y^2)dx+(x^2+xy)dy=0$.

Q16. Find the distance of the point (2, 3, 4) from the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$ measured parallel to the plane $3x+2y+2z=5$.

Q17. Evaluate: $\int\left(\frac{\sin 4x-2}{1-\cos 4x}\right)e^{2x}dx$.

Q18. Solve for x: $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x)$, $0<x<\frac{\pi}{2}$.

Q19. Form the differential equation of the family of curve; $y=ae^x+be^{2x}+ce^{3x}$ where *a, b, c* are some arbitrary constants.

Q20. Probabilities of solving a specific problem in an examination independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved (ii) exactly one of them solves the problem.
"Better time management may help in fetching good marks in the examinations." Comment in one line.

Q21. Using properties of determinants, prove that: $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$.

Q22. If $f(x)=\dfrac{4x+3}{6x-4}, x \neq \dfrac{2}{3}$, show that $fof(x)=x$ for all $x \neq \dfrac{2}{3}$. Write the expression for f^{-1}.

SECTION – C

Q23. Find the area lying above X-axis and included between the circle $x^2+y^2=2ax$ and parabola $y^2=ax$.

OR Draw the rough sketch of the region enclosed between the circles $x^2+y^2=4$ and $(x-2)^2+y^2=1$. Using integration, find the area of the enclosed region.

Q24. Let A be a square matrix. Then show that:

(i) $\dfrac{1}{2}(A+A')$ is a symmetric matrix and, (ii) $\dfrac{1}{2}(A-A')$ is a skew-symmetric matrix.

Also prove that any square matrix can be uniquely expressed as the sum of a symmetric matrix and skew-symmetric matrix.

Q25. Prove that all the normals to the curve $x=p\cos t+pt\sin t,\ y=p\sin t-pt\cos t$ are at a distance of $p\ units$ from the origin.

OR A 20m steel wire is to be cut into two pieces. The first piece is transformed into a circle and the other one into an equilateral triangle. Find out what should be the length of two pieces so that the combined area of both is minimum?

Q26. Evaluate: $\int_0^{\pi} x \log \sin x dx$.

Q27. Find the equation of the line passing through the point P(4, 6, 2) and the point of intersection of the line $\dfrac{x-1}{3}=\dfrac{y}{2}=\dfrac{z+1}{7}$ and the plane whose equation is $x+y-z-8=0$.

Q28. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?
"Meditation & yoga can be quite beneficial." Comment.

Q29. An aeroplane can carry a maximum of 200 passengers. A profit of Rs.1000 is made on each executive class ticket and a profit of Rs.600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit? "Travel through aeroplanes is time saving but is costly." Elaborate in two points.