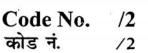Roll No.
रोल नं.

Series DPS

Code No. /2
कोड नं. /2

- Please check that this question paper contains __ printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- Please write down the serial number of the question before attempting it.
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ ___ हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **29** प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें।

MATHEMATICS
गणित

Time allowed : 3 hours
निर्धारित समय : 3 घण्टे

Maximum Marks : 100
अधिकतम अंक : 100

<u>General Instructions</u>: -

i) *All the questions are compulsory.*

ii) *The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 mark each, Section B contains 12 questions of 4 marks each and Section C contains 7 questions of 6 marks each.*

iii) *There is no overall choice. However an internal choice in any 4 questions of four marks each and any two questions of six marks each has been provided.*

iv) *Use of Calculator is not permitted. You may ask for logarithmic tables, if required.*

P.T.O

Section – A

Questions number 1 to 10 carry 1 marks each:

1. Find the order of the differential equation $y^2 = 2c(x + \sqrt{c})$ where c is a parameter.

2. If $f(x) = \sin(7x)°$, find $\dfrac{dy}{dx}$.

3. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear. .

4. Write the direction cosines and direction ratios of the vector $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$.

5. Find the value of p if the angle between the pair of lines $\dfrac{x+3}{3} = \dfrac{y-1}{5p} = \dfrac{z+5}{4}$ and $\dfrac{x+1}{1} = \dfrac{y-4}{1} = \dfrac{z-5}{2}$ is $90°$.

6. Find x, y, z and a for which $\begin{pmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 3 & 2a \end{pmatrix}$.

7 If A is the square matrix of order 3 such that $|adjA| = 64$. Find $|A'|$.

8. If points (2, 0), (0, 5) and (x, y) are collinear, then show that $\dfrac{x}{2} + \dfrac{y}{5} = 1$.

9. Evaluate $\sin[\cos^{-1}(1/2)]$.

10. Find the principal value of $\cot^{-1}\left(\dfrac{-1}{\sqrt{3}}\right)$.

Section-B

Questions number 11 to 22 carry 4 marks each.

11. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

12. Find the equation of the plane that contains the point (1, -1, 2) and is perpendicular to each of the planes 2x + 3y -2z = 5 and x + 2y -3z = 8.

OR

Find the shortest distance between the lines l_1 and l_2 given by
$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \}(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \sim(2\hat{i} + 3\hat{j} + 6\hat{k})$

13. The scalar product of a vector with the vectors $\hat{i} + \hat{j} - 3\hat{k},\ \hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5 and 8 respectively. Find the vector.

OR

Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

14. Evaluate $\displaystyle\int \dfrac{x^4 dx}{(x-1)(x^2-1)}$.

15. Evaluate $\displaystyle\int_0^{f/2} \dfrac{\sin^2 x}{\sin x + \cos x}\, dx$

16. Evaluate $\displaystyle\int \frac{x^2}{(x\sin x + \cos x)^2}\,dx$

17. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}+1}{x}\right)$ with respect to $\cos^{-1}\sqrt{\dfrac{1+x^2}{2}}$.

18. If $f(x) = \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1}$, $x \neq \dfrac{f}{4}$, find the value of $f\left(\dfrac{f}{4}\right)$ so that $f(x)$ become continuous.

19. If $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$, find $\dfrac{d^2 y}{dx^2}$ at $\theta = \dfrac{f}{2}$.

20. By using the properties of determinants prove that :

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right)$$

OR

Using properties of determinants show that:

$$\begin{vmatrix} c^2 b^2 & bc & b+c \\ c^2 a^2 & ca & c+a \\ a^2 b^2 & ab & a+b \end{vmatrix} = 0$$

21. Prove that: $\tan^{-1}\left(\dfrac{1}{x+y}\right) + \tan^{-1}\left(\dfrac{y}{x^2+xy+1}\right) = \cot^{-1} x$

OR

Prove that: $\cos^{-1}\sqrt{\dfrac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}} = \dfrac{1}{2}\tan^{-1} x$

22. Consider $f : R^+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$ show that f is invertible, also find f^{-1}.

Section-C

Question number 23 to 29 carry 6   marks each

23. Using matrices, solve the following system of equations:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

OR

Obtain the inverse of the following matrix using elementary operations: $\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$.

24. An open tank with a square base and vertical sides is to be constructed from a metal sheet to hold a given quantity of water. Show that the cost of the material will be least when the depth of the tank is half of its width.

25. Find the area of the region : $\{(x, y) : 0 \le y \le x^2; 0 \le y \le x + 2; 0 \le x \le 3\}$.

26. Find the particular solution of the differential equation $\left[x \ \sin^2 \dfrac{y}{x} - y \right] dx + x \ dy = 0; \ y = \dfrac{f}{4}$ when x=1 .

27. Find the equation of the plane through the point (1,1,1) and containing the line
$\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \}(3\hat{i} - \hat{j} + 5\hat{k})$.

Also, show that the plane contains the line $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \}(\hat{i} - 2\hat{j} - 5\hat{k})$.

28. A toy company manufacturers two type of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 12000 dolls per week and the demand of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of doll of other type by at most 6000 units. If the company makes profit of Rs120 and Rs.160 per doll respectively on doll A and B, how many of each should be produced weekly in order to maximise the profit?

29. In an examination, 10 questions of true-false type are asked. A girl tosses a fair coin to determine her answer to each question. If the coin falls heads, she answers 'true' and if it falls tails she answers 'false'.

Show that the probability that she answers at most 7 questions correctly is $\dfrac{121}{128}$.

OR

In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices and only one correct option. The probability that he guess is $\dfrac{1}{3}$ .The probability that he copies the answer is $\dfrac{1}{6}$ .The probability that the answer is correct, given that he copied, is $\dfrac{1}{8}$. Find the probability that he knows the answer to the question, given that he correctly answered it.