

1. Find the LCM and HCF of the following pairs of integers and verify that LCM X HCF = product of integers

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Solution-

$$\begin{array}{r}
 26 \overline{) 91} \quad (\quad 3 \\
 \underline{78} \\
 13 \overline{) 26} \quad (\quad 2 \\
 \underline{26} \\
 0
 \end{array}
 \qquad
 \begin{array}{r}
 13 \overline{) 26} \quad \text{--} \quad 91 \\
 \underline{26} \quad \text{--} \quad 7 \\
 \hline
 \end{array}$$

$$\text{HCF} = 13$$

$$\text{LCM} = 13 \times 2 \times 7 = 182$$

$$\text{1st number} \times \text{2nd number} = \text{HCF} \times \text{LCM} \quad 26 \times 91 = 13 \times 182 = 2366$$

2. Find the LCM and HCF of the following integers by applying the prime factorization method

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25 (iv) 40, 36 and 126
(v) 84, 90 and 126 (vi) 24, 15 and 36

Solution- (i) $12 = 2 \times 2 \times 3$ **HCF** = 3

$$15 = 3 \times 5 \qquad \text{LCM} = 2 \times 2 \times 3 \times 5 \times 7 = 720$$

$$21 = 3 \times 7$$

3. Find the greatest number of 6-digit exactly divisible by 24, 15 and 36

Solution- The greatest 6-digit number is 999999

To find the LCM of 15, 24, 36

$$\begin{array}{r}
 2 \overline{) 15, 24, 36} \\
 \hline
 2 \overline{) 15, 12, 18} \\
 \hline
 3 \overline{) 15, 6, 9} \\
 \hline
 5, 2, 3
 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 3 \times 5 \times 2 \times 3 = 360$$

$$\text{The greatest 6-digit number is } 999999 - 360 = 999720$$

4. A rectangular courtyard is 18m72cm long and 13m20cm broad. It is to be paved

With square tiles of same size. Find the least possible numbers of such tiles

Solution- length=1872cm breadth= 1320cm

HCF of 1872 and 1320 is 24

$$\text{No. of tiles} = \frac{\text{area of courtyard}}{\text{area of one tile}} = \frac{1872 \times 1320}{24} = 4290$$

5. Find the least numbers that is divisible by all the numbers between 1 and 10

(both inclusive)

Solution- Find the LCM of 1,2,3,4,5,6,7,8,9,10

$$\begin{array}{r}
 2 \quad 1,2,3,4,5 \mid 6,7,8,9,10 \\
 \hline
 2 \quad 1,1,3,2,5 \mid 3,7,4,9,5 \\
 \hline
 2 \quad 1,1,3,1,5 \mid 3,7,2,9,5 \\
 \hline
 3 \quad 1,1,1,1,5 \mid 1,7,2,3,5 \\
 \hline
 5 \quad 1,1,1,1,1 \mid 1,7,2,3,1 \\
 \hline
 \text{LCM} = 2 \times 2 \times 2 \times 3 \times 5 \times 7 \times 2 \times 3 = 2520
 \end{array}$$

This is the required number

6. What is the smallest number that when divided by 35,56 and 91 leaves remainders 7 of in each case?.

Solution- The LCM of 35,56 and 91

$$\begin{array}{r}
 7 \quad 35 \mid 56, 91 \\
 \hline
 5 \quad 8 \mid 13 \\
 \hline
 \text{LCM} = 7 \times 5 \times 8 \times 13 = 3640
 \end{array}$$

Hence required number is $3640+7=3647$

7. In morning walk three persons step off together ,their steps measure 80cm ,85cm and 90cm respectively . what is the minimum distance each should walk so

that he can cover the distance in complete steps ?

Solution- The LCM of 80,85 and 90

$$\begin{array}{r|l}
 2 & 80, 85, 90 \\
 \hline
 5 & 40, 85, 45 \\
 \hline
 5 & 17, 9
 \end{array}$$

$$\text{LCM} = 2 \times 5 \times 5 \times 17 \times 9 = 12240 = 122\text{m}40\text{cm}$$

8. Determine the number nearest to 110000 but greater than 100000 which is exactly divisible by 8,15 and 21

Solution- The LCM of 8,15 and 21 now $110000 \div 840$

$$\begin{array}{r|l}
 3 & 8, 15, 21 \\
 \hline
 & 8, 5, 7
 \end{array}$$

LCM = $3 \times 8 \times 5 \times 7 = 840$

Remainder = 800
The required no. is $= 110000 - 800 = 109200$

9. find the smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively

Solution- The LCM of 28 and 32

$$\begin{array}{r|l}
 2 & 28, 32 \\
 \hline
 2 & 14, 16 \\
 \hline
 & 7, 8
 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 7 \times 8 = 224$$

But remainders are 8 and 12

$$\text{So required number} = 224 - 20 = 204$$

10. Find the smallest number which when increased by 17 is exactly divisible by

both 520 and 468

Solution- The LCM of 520 and 468

$$\begin{array}{r|l}
 2 & 468, 520 \\
 \hline
 2 & 234, 260 \\
 \hline
 13 & 117, 130 \\
 \hline
 & 9, 10 \\
 \hline
 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 13 \times 9 \times 10 = 4680$$

$$\text{Required number} = 4680 - 17 = 4663$$

11. A circular field has a circumference of 360km .Three cyclist starts together and can cycle 48,60 and 72km a day, round the field When will they meet again?

12. The LCM and HCF of two numbers are 180 and 6 respectively . If one of the numbers is 30, find the other number.

Solution- $1^{\text{st}} \text{ no.} \times 2^{\text{nd}} \text{ no.} = \text{LCM} \times \text{HCF}$

$$30. \times 2^{\text{nd}} \text{ no.} = 180 \times 6$$

$$2^{\text{nd}} \text{ no.} = \frac{180 \times 6}{30} = 36$$

13.The HCF of two numbers is 16 and their product is 3072 .Find their LCM

Solution- $1^{\text{st}} \text{ no.} \times 2^{\text{nd}} \text{ no.} = \text{LCM} \times \text{HCF}$

$$3072 = \text{LCM} \times 16$$

$$\text{LCM} = \frac{3072}{16} = 192$$

14. The HCF of two numbers is 145 and their LCM is 2175. If one of the

numbers is 725, find the other number.

Solution- $1^{\text{st}} \text{ no.} \times 2^{\text{nd}} \text{ no.} = \text{LCM} \times \text{HCF}$

$$725 \times 2^{\text{nd}} \text{ no.} = 2175 \times 145$$

$$2^{\text{nd}} \text{ no.} = \frac{2175 \times 145}{725} = 435$$

15. Can two numbers have 16 as their HCF and 380 as their LCM? Give reason

Solution- No, because 380 is not divisible by 16

POLYNOMIALS

EXERCISE-2.1

Q1- (i)

$$x^2 - 2x - 8$$

$$x^2 + (-4+2)x - 8$$

$$(x^2 - 4x) + (2x - 8)$$

$$x(x-4) + 2(x-4) = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4, -2$$

$$\text{Sum} = 4 + (-2) = 2$$

$$\text{Product} = 4 \times -2 = -8$$

$$\text{sum} = \frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2} = \frac{-(-2)}{1} = 2$$

$$\text{product} = \frac{\text{constant term}}{\text{co-efficient of } x^2} = \frac{-8}{1} = -8$$

$$(v) \sqrt{3}x^2 + 10x + 7\sqrt{3}$$

$$\sqrt{3}x^2 + (7+3)x + 7\sqrt{3}$$

$$(\sqrt{3}x^2 + 7x) + (3x + 7\sqrt{3}) = 0$$

$$\sqrt{3}x(x + \frac{7}{\sqrt{3}}) + 3(x + \frac{7\sqrt{3}}{3} x \frac{\sqrt{3}}{\sqrt{3}}) = 0$$

$$x = -\frac{7}{\sqrt{3}} \quad x = \frac{-3}{\sqrt{3}}$$

$$\text{Sum} = \frac{-7-3}{\sqrt{3}} = \frac{-10}{\sqrt{3}}$$

$$\text{Product} = -\frac{7}{\sqrt{3}} \times \frac{-3}{\sqrt{3}} = \frac{21}{3} = 7$$

$$\text{sum} = \frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2} = \frac{-(10)}{\sqrt{3}} = \frac{-10}{\sqrt{3}}$$

$$\text{product} = \frac{\text{constant term}}{\text{co-efficient of } x^2} = \frac{7\sqrt{3}}{\sqrt{3}} = 7$$

$$(vi) x^2 - (\sqrt{3} + 1)x + \sqrt{3}$$

$$(x^2 - \sqrt{3}x) - (x + \sqrt{3})$$

$$x(x - \sqrt{3}) - 1(x + \sqrt{3})$$

$$(x - \sqrt{3})(x - 1) = 0$$

$$x = \sqrt{3}, 1$$

$$\text{Sum} = \sqrt{3} + 1$$

$$\text{Product} = \sqrt{3} \times 1 = \sqrt{3}$$

$$\text{sum} = \frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2} = \frac{-\{-(\sqrt{3} + 1)\}}{1} = \sqrt{3} + 1$$

$$\text{product} = \frac{\text{constant term}}{\text{co-efficient of } x^2} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$(vii) a(x^2 + 1) - x(a^2 + 1)$$

$$ax^2 + a - xa^2 - x$$

$$ax^2 - (a^2 + 1)x + a$$

$$(ax^2 - xa^2) - (x + a)$$

$$ax(x - a) - 1(x + a) = 0$$

$$(x - a)(ax - 1) = 0$$

$$x = a, \frac{1}{a}$$

$$\text{Sum} = \frac{a^2 + 1}{a}$$

$$\text{Product} = a \times \frac{1}{a} = 1$$

$$\text{sum} = \frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2} = \frac{-\{-(a^2 + 1)\}}{a}$$

$$= \frac{a^2 + 1}{a}$$

$$\text{product} = \frac{\text{constant term}}{\text{co-efficient of } x^2} = \frac{a}{a} = 1$$

Part no. (ii) (iii) and (iv) are similar

Q2- if α and β are the zeros of quadratic polynomials ax^2+bx+c then evaluate

$$(i) \alpha - \beta \quad (iii) \frac{1}{\alpha} - \frac{1}{\beta} - 2\alpha\beta \quad (iv) \alpha^2\beta + \alpha\beta^2 \quad (v) \alpha^4 + \beta^4 \quad (vi) \frac{1}{a\alpha+b} + \frac{1}{a\beta+b}$$

$$(ii) \frac{1}{\alpha} - \frac{1}{\beta} \quad (vii) \frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$$

Solution- the zeros of quadratic polynomial is

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha - \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} - \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(-b + \sqrt{b^2 - 4ac}) - (-b - \sqrt{b^2 - 4ac})}{2a}$$

$$= \frac{-b + \sqrt{b^2 - 4ac} + b + \sqrt{b^2 - 4ac}}{2a} = \frac{2\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{\sqrt{b^2 - 4ac}}{a}$$

$$(ii) \frac{\alpha - \beta}{\alpha\beta}$$

$$\alpha - \beta = \frac{\sqrt{b^2 - 4ac}}{a}$$

$$\alpha\beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{4a^2}$$

$$\frac{\cancel{b^2} - \cancel{b^2} - 4ac}{4\cancel{a}a} = \frac{c}{a}$$

$$\frac{\alpha - \beta}{\alpha\beta} = \frac{\sqrt{b^2 - 4ac}}{a} \quad \frac{c}{a}$$

$$\frac{\sqrt{b^2 - 4ac}}{a} \quad \frac{a}{c}$$

$$(iii) \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta \quad \frac{\alpha + \beta}{\alpha\beta} - 2\alpha\beta = \frac{(-b)/a}{c/a} - 2\frac{c}{a} = -\left\{\frac{b}{c} + \frac{2c}{a}\right\}$$

c/a

$$(iv) \alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = \frac{c}{a} \times \frac{-b}{a} = \frac{-bc}{a^2}$$

(v) $\alpha^4 + \beta^4$ Solve on page no.229 example 8 RDSA HARMA

$$(vi) \frac{1}{a\alpha + b} + \frac{1}{a\beta + b} = \frac{a\beta + b + a\alpha + b}{(a\alpha + b)(a\beta + b)} = \frac{a(\alpha + \beta) + 2b}{a^2(\alpha\beta) + ab(\alpha + \beta) + b^2}$$

$$= \frac{a\left(\frac{-b}{a}\right) + 2b}{a^2\left(\frac{c}{a}\right) + ab\left(\frac{-b}{a}\right) + b^2} = \frac{b}{ac} \left\{ \alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a} \right\}$$

$$(vii) \frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{\beta(a\beta+b) + \alpha(a\alpha+b)}{(a\alpha+b)(a\beta+b)} = \frac{\beta(a\beta+b) + \alpha(a\alpha+b)}{(a\alpha+b)(a\beta+b)}$$

$$= \frac{a(\alpha^2 + \beta^2) + b(\alpha + \beta)}{a^2(\alpha\beta) + ab(\alpha + \beta) + b^2}$$

$$= (\alpha^2 + \beta^2) - (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \frac{a(\alpha + \beta)^2 - 2\alpha\beta + b(\alpha + \beta)}{a^2(\alpha\beta) + ab(\alpha + \beta) + b^2} = \frac{a\left(\frac{-b}{a}\right)^2 - 2\frac{c}{a} + b\left(\frac{-b}{a}\right)}{a^2\left(\frac{c}{a}\right) + ab\left(\frac{-b}{a}\right) + b^2} = \frac{-2}{a^2}$$

Q3- $6x^2 + x - 2$ evaluate $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Solution- $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

sum = $\frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2}$, product = $\frac{\text{constant term}}{\text{co-efficient of } x^2}$

$$\alpha + \beta = \frac{-1}{6} \quad \alpha\beta = \frac{-2}{6} = \frac{-1}{3}$$

$$\frac{\left(\frac{-1}{6}\right)^2 - 2\left(\frac{-1}{3}\right)}{\frac{-1}{3}} = \frac{-25}{12}$$

Q4- $x^2 - x - 4$, evaluate $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$

Solution- sum = $\frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2}$, product = $\frac{\text{constant term}}{\text{co-efficient of } x^2}$

$$\alpha + \beta = \frac{-(-1)}{1} = 1 \quad \alpha\beta = \frac{-4}{1} = -4$$

$$= \frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$$

$$= \frac{\alpha + \beta}{\alpha\beta} - \alpha\beta = \frac{1}{-4} - (-4) = \frac{15}{4}$$

Q5- $p(x) = 4x^2 - 5x - 1$ evaluate $\alpha^2\beta + \alpha\beta^2$

Solution - $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$

$$\text{sum} = \frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2}, \text{ product} = \frac{\text{constant term}}{\text{co-efficient of } x^2}$$

$$\alpha + \beta = \frac{-(-5)}{4} \quad \alpha\beta = \frac{-1}{4}$$

$$= \alpha\beta(\alpha + \beta) = \frac{5}{4} \left(\frac{-1}{4} \right) = \frac{-5}{16}$$

Q6- $f(x) = x^2 + x - 2$ evaluate $\frac{1}{\alpha} - \frac{1}{\beta}$

$$\text{Solution- } \frac{1}{\alpha} - \frac{1}{\beta} = \frac{\alpha - \beta}{\alpha\beta}$$

$$\text{Sum} = \frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2}, \text{ product} = \frac{\text{constant term}}{\text{co-efficient of } x^2}$$

$$\alpha + \beta = \frac{-1}{1} = -1, \quad \alpha\beta = \frac{-2}{1} = -2$$

$$\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$= \sqrt{(-1)^2 - 4(-2)}$$

$$= \sqrt{9} = 3$$

$$= \frac{\alpha - \beta}{\alpha\beta} = \frac{3}{-2}$$

Q7- $x^2 - 5x + 4$ evaluate $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

Solution- $\text{sum} = \frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2}$, $\text{product} = \frac{\text{constant term}}{\text{co-efficient of } x^2}$

$$= \frac{\alpha + \beta}{\alpha\beta} - 2\alpha\beta$$

$$\alpha + \beta = \frac{-(-5)}{1} = 5 \quad \alpha\beta = \frac{4}{1} = 4$$

$$= \frac{5}{4} - 2(4) = \frac{-27}{4}$$

Q8- $f(t) = t^2 - 4t + 3$ evaluate $\alpha^4\beta^3 + \alpha^3\beta^4$

Solution - $\text{sum} = \frac{-(\text{co-efficient of } t)}{\text{co-efficient of } t^2}$, $\text{product} = \frac{\text{constant term}}{\text{co-efficient of } t^2}$

$$\alpha + \beta = \frac{-(-4)}{1} = 4 \quad \alpha\beta = \frac{3}{1} = 3$$

$$= \alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta) = (\alpha\beta)^3(\alpha + \beta)$$

$$= (4)^3(3) = 192$$

Q9- $p(y) = 5y^2 - 7y + 1$ evaluate, $\frac{1}{\alpha} + \frac{1}{\beta}$

Solution - $\text{sum} = \frac{-(\text{co-efficient of } y)}{\text{co-efficient of } y^2}$, $\text{product} = \frac{\text{constant term}}{\text{co-efficient of } y^2}$

$$\alpha + \beta = \frac{-(-7)}{5} = \frac{7}{5} \quad \alpha\beta = \frac{1}{5}$$

$$\frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{7}{5}}{\frac{1}{5}} = 7$$

Q10- $p(s) = 3s^2 - 6s + 4$ evaluate, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left\{\frac{1}{\alpha} + \frac{1}{\beta}\right\} + 3\alpha\beta$

Solution - $\text{sum} = \frac{-(\text{co-efficient of } s)}{\text{co-efficient of } s^2}$, $\text{product} = \frac{\text{constant term}}{\text{co-efficient of } s^2}$

$$\alpha + \beta = \frac{-(-6)}{3} = 2 \quad \alpha\beta = \frac{4}{3}$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2 \left\{ \frac{\alpha + \beta}{\alpha\beta} \right\} + 3\alpha\beta$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2 \left\{ \frac{\alpha + \beta}{\alpha\beta} \right\} + 3\alpha\beta = \frac{(2)^2 - 2 \times \frac{4}{3}}{\frac{4}{3}} + 2 \left\{ \frac{2}{\frac{4}{3}} \right\} + 3 \times \frac{4}{3} = 8$$

Q11- $x^2 - px + q$ prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

Solution - sum = $\frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2}$, product = $\frac{\text{constant term}}{\text{co-efficient of } x^2}$

$$\alpha + \beta = \frac{-(-p)}{1} = p \quad \alpha\beta = \frac{q}{1} = q$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (p)^2 - 2q = p^2 - 2q$$

$$\alpha^2\beta^2 = q^2$$

$$= \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2} = \frac{(\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2} = \frac{(p^2 - 2q)^2 - 2q^2}{q^2}$$

$$\frac{(p^4 + 4q^2 - 4p^2q) - 2q^2}{q^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$$

Q12-

Q13-if the sum of zeros of quadratic polynomials is $f(t) = kt^2 + 2t + 3k$ is equal to their product, find the value of k

Solution - sum = $\frac{-(\text{co-efficient of } t)}{\text{co-efficient of } t^2}$, product = $\frac{\text{constant term}}{\text{co-efficient of } t^2}$

$$\alpha + \beta = \frac{-(2)}{k} = \frac{-2}{k} \quad \alpha\beta = \frac{3k}{k} = 3$$

According to statement

$$\alpha + \beta = \alpha\beta$$

$$\frac{-2}{k} = 3, \quad k = \frac{-2}{3}$$

Q14-

Q15-if α and β are zeros of quadratic polynomial $f(x)=x^2-1$, find a quadratic polynomial whose zeros are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$

Solution - sum = $\frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2}$, product = $\frac{\text{constant term}}{\text{co-efficient of } x^2}$

$$\alpha + \beta = \frac{-(0)}{1} = 0 \quad \alpha\beta = \frac{-1}{1} = -1$$

$$\text{sum} = \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{2\alpha^2 + 2\beta^2}{\alpha\beta} = \frac{2(\alpha^2 + \beta^2)}{\alpha\beta} = \frac{2\{(\alpha + \beta)^2 - 2\alpha\beta\}}{\alpha\beta} = \frac{2\{(0)^2 - 2(-1)\}}{-1} = -4$$

$$\text{product} = \frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha} = 4$$

Required polynomial is $f(x) = k(x^2 - sx + p)$

$$= k(x^2 - (-4)x + 4)$$

$$= k(x^2 + 4x + 4)$$

Q16- if α and β are zeros of quadratic polynomial $f(x)=x^2-3x-2$, find a quadratic polynomial whose zeros are $\frac{1}{2\alpha+\beta}$, $\frac{1}{2\beta+\alpha}$

Solution - sum = $\frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2}$, product = $\frac{\text{constant term}}{\text{co-efficient of } x^2}$

$$\alpha + \beta = \frac{-(-3)}{1} = 3 \quad \alpha\beta = \frac{-2}{1} = -2$$

$$\text{sum} = \frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha} = \frac{2\beta+\alpha+2\alpha+\beta}{(2\alpha+\beta)(2\beta+\alpha)} = \frac{3\alpha+3\beta}{(4\alpha\beta+2\beta^2+2\alpha^2+\alpha\beta)} = \frac{3(\alpha+\beta)}{(5\alpha\beta+2(\alpha^2+\beta^2))}$$

$$\frac{3(\alpha+\beta)}{\{(5\alpha\beta+2(\alpha+\beta)^2-2\alpha\beta)\}} = \frac{3(3)}{\{(5X-2+2\{(3)^2-2X-2)\}} = \frac{9}{16}$$

$$\begin{aligned} \text{Product} &= \frac{1}{2\alpha+\beta} X \frac{1}{2\beta+\alpha} = \frac{1}{(2\alpha+\beta)(2\beta+\alpha)} = \frac{1}{(4\alpha\beta+2\beta^2+2\alpha^2+\alpha\beta)} = \frac{3(\alpha+\beta)}{(5\alpha\beta+2(\alpha^2+\beta^2))} \\ &= \frac{1}{\{(5\alpha\beta+2(\alpha+\beta)^2-2\alpha\beta)\}} = \frac{1}{\{(5X-2+2\{(3)^2-2X-2)\}} = \frac{1}{16} \end{aligned}$$

Required polynomial is $f(x) = k(x^2 - \frac{9}{16}x + \frac{1}{16})$

Q17-- if α and β are zeros of quadratic polynomial such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$ find a quadratic polynomial whose zeros are α and β

Solution - $\alpha + \beta = 24$

$$\alpha - \beta = 8$$

$$2\alpha = 32, \alpha = \frac{32}{2} = 16, \beta = 24 - 16 = 8, \alpha\beta = 16 \times 8 = 128$$

Required polynomial is $f(x) = k(x^2 - sx + p)$

$$= k(x^2 - 24x + 128)$$

Q18- if α and β are zeros of quadratic polynomial $f(x) = x^2 - p(x+1) - c$ then show that

$$(\alpha + 1)(\beta + 1) = 1 - c$$

Solution - $x^2 - px - p - c$

$$= x^2 - px - (p+c)$$

$$\text{Sum} = \frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2}, \quad \text{product} = \frac{\text{constant term}}{\text{co-efficient of } x^2}$$

$$\alpha + \beta = \frac{-(-p)}{1} = p, \quad \alpha\beta = \frac{-(p+c)}{1} = -(p+c)$$

$$(\alpha + 1)(\beta + 1) = (\alpha\beta + (\alpha + \beta) + 1) = -(p + c) + (p) + 1 = 1 - c$$

Q19- if α and β are zeros of quadratic polynomial $f(x) = x^2 - 2x + 3$, find a quadratic polynomial whose zeros are (i) $\alpha + 2, \beta + 2$ (ii) $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

Solution – Sum = $\frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2}$, product = $\frac{\text{constant term}}{\text{co-efficient of } x^2}$

$$\alpha + \beta = \frac{-(-2)}{1} = 2, \quad \alpha\beta = \frac{3}{1} = 3$$

(i) sum = $\alpha + 2 + \beta + 2 = (\alpha + \beta) + 4 = 2 + 4 = 6$

product = $(\alpha + 2)(\beta + 2) = \alpha\beta + 2(\alpha + \beta) + 4 = 3 + 2(2) + 4 = 11$

Required polynomial is $f(x) = k(x^2 - sx + p)$

$$= k(x^2 - 6x + 11)$$

(ii) $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

$$\text{Sum} = \frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1} = \frac{(\alpha-1)(\beta+1) + (\alpha+1)(\beta-1)}{(\alpha+1)(\beta+1)} = \frac{\alpha\beta - \beta + \alpha - 1 + \alpha\beta + \beta - \alpha - 1}{(\alpha\beta + (\alpha + \beta) + 1)}$$

$$= \frac{2\alpha\beta - 2}{(\alpha\beta + (\alpha + \beta) + 1)} = \frac{2 \times 3 - 2}{(2 + (3) + 1)} = \frac{4}{6} = \frac{2}{3}$$

$$\text{Product} = \frac{\alpha-1}{\alpha+1} \times \frac{\beta-1}{\beta+1} = \frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)} = \frac{(\alpha\beta - \beta - \alpha - 1)}{(\alpha\beta + \beta + \alpha + 1)} = \frac{(\alpha\beta - \{\beta + \alpha\} - 1)}{(\alpha\beta + \{\beta + \alpha\} + 1)} = \frac{(3 - \{2\} - 1)}{(3 + \{2\} + 1)}$$

$$\frac{2}{6} = \frac{1}{3}$$

Required polynomial is $f(x) = k(x^2 - \frac{2}{3}x + \frac{1}{3})$

Q20- if α and β are zeros of quadratic polynomial $f(x)=x^2+px+q$, find a quadratic polynomial whose zeros are (i) $(\alpha + \beta)^2, (\alpha - \beta)^2$

Solution – Sum = $\frac{-(\text{co-efficient of } x)}{\text{co-efficient of } x^2}$, product = $\frac{\text{constant term}}{\text{co-efficient of } x^2}$

$$\alpha + \beta = \frac{-(p)}{1} = -p, \quad \alpha\beta = \frac{q}{1} = q$$

$$\begin{aligned} \text{sum} &= (\alpha + \beta)^2 + (\alpha - \beta)^2 = (\alpha^2 + \beta^2 + 2\alpha\beta) + (\alpha^2 + \beta^2 - 2\alpha\beta) = 2(\alpha^2 + \beta^2) \\ &= 2\{p^2 - 2q\} \end{aligned}$$

$$\text{Product} = (\alpha + \beta)^2 \times (\alpha - \beta)^2 = (\alpha^2 + \beta^2 + 2\alpha\beta)(\alpha^2 + \beta^2 - 2\alpha\beta)$$

$$= \{(\alpha + \beta)^2 - 2\alpha\beta + 2\alpha\beta\} \{(\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta\}$$

$$= \{(\alpha + \beta)^2\} \{(\alpha + \beta)^2 - 4\alpha\beta\}$$

$$= \{(-p)^2\} \{(-p)^2 - 4q\}$$

$$= p^2 \{p^2 - 4q\}$$

= Required polynomial is $f(x) = k(x^2 - sx + p)$

$$= k\{x^2 - 2(p^2 - 2q)x + p^2 - 4q\}$$

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IF YOU WANT WHOLE SA-1 UNSOLVED EXERCISE OF SA-1

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