SUNIL TUTORIALS

For Classes Xth, XIth, XIIth, & B.Com (H/P)
(Mathematics, Accountancy, Taxation, Economics., English, Costing & S
F-3/713, Sangam Vihar, New Delhi –110062.

Exercise 1

Q.1. Find sum A+B of the following matrices:

(i) $A=\begin{bmatrix}2 & -1\\ 3 & 5\end{bmatrix};B=\begin{bmatrix}4 & 3\\ 1 & -2\end{bmatrix}$

(ii) $A=\begin{bmatrix}\sin^2\theta & -\cos^2\theta\\ 1 & \sec^2\theta\end{bmatrix};B=\begin{bmatrix}\cos^2\theta & -\sin^2\theta\\ 0 & -\tan^2\theta\end{bmatrix}$

(iii) $A=\begin{bmatrix}a^2+b^2 & b^2+c^2\\ a^2+c^2 & a^2+b^2\end{bmatrix};B=\begin{bmatrix}2ab & 2bc\\ -2ac & -2ab\end{bmatrix}$

Q.2. If $A=\begin{bmatrix}2 & -1\\ 4 & 2\end{bmatrix};B=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix};C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Find each of the following:

(i) $A+B+C$; (ii) $A+B-C$; (iii) $2A-C$; (iv) $3A+B$; (v) $2A-3B$

Q.3. (a) If matrices $\begin{bmatrix}a+b & 2\\ 5 & ab\end{bmatrix}=\begin{bmatrix}6 & 2\\ 5 & 8\end{bmatrix}$. Find the value of a and b

(b) Find x, y, z, t if $2\begin{bmatrix}x & z\\ y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\ 4 & 6\end{bmatrix}$

Q.4. (a) Construct a 2×2 matrix $B=[b_{ij}]$ whose elements are given below:

(i) $b_{ij}=\frac{(i-2j)^2}{2}$ (ii) $b_{ij}=ij$ (iii) $b_{ij}=\frac{1}{2}|2i-3j|$

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Q.5. Find x and y if

(a) $2X+Y=\begin{bmatrix}4 & 4 & 7\\ 7 & 3 & 4\end{bmatrix}$ and $X-2Y=\begin{bmatrix}-3 & 2 & 1\\ 1 & -1 & 2\end{bmatrix}$

(b) $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

Q.6. If $A=\begin{bmatrix}3 & 2 & 2\\ -3 & -1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & 1\\ 2 & 1 & 3\end{bmatrix}$

Find the matrix C such that A+B+C is a zero matrix.

Q.7. If $A = diag[3,5,-7]$ and $B = diag[2,-3,5]$
Find A+2B and 3B-A

Q.8. If $A=\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}; B=\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$, find the matrix X such that 2X+3A-4B=0.

Q.9. Q.1 Find a matrix C such that A+B+C is a zero matrix.

$$A=\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} and\ B=\begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Q.10. Construct a 2×3 matirx whose elements a_{ij} are given by $a_{ij}=2i-j$

Q.11. Find a matrix X such that 2A+B+X=0 where $A=\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}, B=\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

Q.12. If $\begin{bmatrix} x+y & 3x-2y \\ 5x-z & 2y-1 \end{bmatrix}=\begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}$, find x, y, z.

Q.13. Find X if 2X+Y= $\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$ and Y=$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Q.14. Write a martix $(a_{ij})_{3\times 4}$ in which $a_{ij}=\begin{cases} 2i+j\ when\ i<j \\ 4ij \quad when\ i=j \\ i+2j\ when\ i>j \end{cases}$

Exercise 2

Q.1. Find the product AB of the following matrices:

(i) $A=\begin{bmatrix} 0 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}; B=\begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix}$ (ii) $A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}; B=\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$

(iii) $A=\begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}; B=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

Q.2. If $A=\begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}$ show that $A^2=A$

Q.3. Evaluate: $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}+2\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

Q.4. Find the value of x such that $\begin{bmatrix} 1 & 1 & x \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=0$

Q.5. If $A=\begin{bmatrix} 3 & 2 \\ 12 & 8 \end{bmatrix}$ and $B=\begin{bmatrix} 8 & 4 \\ -12 & -6 \end{bmatrix}$ show that AB=0.

Q.6. If $A=\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}; B=\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ and $C=\begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$, verify that (AB)C=A(BC) and A(B+C)=AB+AC.

Q.7. Show that $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^3=\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

Q.8. Evaluate: $\left\{\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}+\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix}\right\}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Q.9. If $A=\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, Show that $A^2-3I=2A$.

Q.10. Let $f(x)=x^2-5x+6$, find $f(A)$ if $A=\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Q.11. If $F(x)=\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$,then show that $F(x)F(y)=F(x+y)$.

Q.12. (a) If a matrix $A=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, prove that $A^k=\begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$.

(b) If $A=\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, prove that $A^n=\begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

Q.13. Without using the concept of inverse of a matrix $\begin{bmatrix} x & y \\ z & u \end{bmatrix}$, find the matrix such that

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x & y \\ z & u \end{bmatrix}=\begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}.$$

Q.14. If $A=\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, find $A^2-5A-14I$

Q.15. Mr. Sunil has Rs. 30,000 and he wants to invest in two different types of bounds. The first bond pays 5% interest per annum, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs. 30,000 among the two types of bonds so that the fund must obtain an annual total interest of (i) 1,800; (ii) Rs. 2,000.

Q.16. Three shop keepers Ram, Mohinder and Sham go to a store to buy stationery. Shopkeeper Ram purchases 12 dozen notebooks, 5 dozen pens, 6 dozen pencils.Shopkeeper Mohinder purchases

10 dozen notebooks, 6 dozen pens and 7 dozen pencils. Shopkeeper Sham purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs 40 paise, a pen costs Rs. 1.25 and a pencil costs 35 paise each. Use matrix multiplication to calculate each individual's bill.

Q.17. The Co-operative store of a particular central school has in stock 10 dozen Mathematics IX books, 8 dozen Mathematics X books and 5 dozen Insight into Mathematics V dozen. The selling prices are Rs. 8.30, Rs. 3.45 and Rs. 4.50 each, respectively. Find the total amount the store will receive by selling all the items.

Q.18. Solve the matrix equation:

$$\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$$

Q.19. If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$. Find x, y, z, w.

Q.20. Evaluate:

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

Q.21. Let $f(x) = x^2 - 5x + 6$, find $f(A)$ if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Q.22. Find x such that:

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$$

23. If A= $\begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$ and I is a unit matrix. Prove that $(I+A) = (I-A)\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

Exercise 3

Q.1. Write down the transpose of the matrix:

(a) $A = \begin{bmatrix} 1 & 2 & 4 \\ 6 & 8 & 1 \end{bmatrix}$ (b) A= $\begin{bmatrix} -2 & 3 & 4 \\ 5 & -4 & -3 \\ 7 & 2 & 9 \end{bmatrix}$

Q.2. If A=$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$, verify that $\left(A'\right)' = A$

Q.3. If A=$\begin{bmatrix} 2 & 4 \\ -7 & -5 \end{bmatrix}$ and B=$\begin{bmatrix} 1 & -17 \\ 6 & 8 \end{bmatrix}$, verify that $\left(A+B\right)' = A' + B'$

Q.4. If A=$\begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix}$ and B=$\begin{bmatrix} -1 & 0 \\ 2 & 5 \\ 3 & 4 \end{bmatrix}$ find (BA)'

Q.5. Find the values of x, y, z if the matrix A=$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ obeys the law $A^T A = I$

Q.6. (a) Show that A=$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 2 & -1 \\ 5 & -1 & 3 \end{bmatrix}$ is a symmetric matrix.

(b) Show that A=$\begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix}$ is a skew-symmetric matrix.

Q.7. Express A=$\begin{bmatrix} 2 & -4 & 5 \\ 3 & 5 & 2 \\ 7 & 3 & 8 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrix.

to help in the given questions E-mail to us ftcst78@yahoo.com

Q.8. If A is any sqaure matrix, show that
(i) $A+A'$ is symmetric.
(ii) $A-A'$ is skew-symmetric.

Q.9. Express the matrix

$$\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

as the sum of a symmetric and a skew-symmetric matrix.

Q.10. If A=$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, verify that $AA' = A'A = I_2$.

ANSWERS
EXERCISE 1

Q.1. (i) $\begin{bmatrix} 6 & 2 \\ 4 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$

Q.2. (i) $\begin{bmatrix} 5 & 2 \\ 5 & 5 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & -2 \\ 8 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} 8 & 0 \\ 13 & 8 \end{bmatrix}$ (v) $\begin{bmatrix} -2 & -11 \\ 5 & -2 \end{bmatrix}$

Q.3. (a) a = 2, b = 4 or a = 4, b = 2 (b) x = 3, y = 6, z = 9, t = 6

Q.4. (i) $\begin{bmatrix} 1/2 & 9/2 \\ 0 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 1/2 & 2 \\ 1/2 & 1 \end{bmatrix}$ **Q.5** (a) $X=\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}; Y=\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(b) $X=\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}; Y=\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ **Q.6** . $\begin{bmatrix} -2 & -2 & -3 \\ 1 & 0 & -3 \end{bmatrix}$ **Q.7** $\begin{bmatrix} 7 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & 22 \end{bmatrix}$

Q.8. $\begin{bmatrix} -5/2 & 11/2 \\ -9 & -6 \end{bmatrix}$ **Q.9.** $\begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$ **Q.10.** $\begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & -1 \end{bmatrix}$ **Q.11.**

$\begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$ **Q.12.** $x=1, y=2, z=3$ **Q.13.** $\begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$ **Q.14.** $\begin{bmatrix} 4 & 4 & 5 & 6 \\ 4 & 16 & 7 & 8 \\ 5 & 7 & 36 & 10 \end{bmatrix}_{3\times4}$

EXERCISE 2

Q.1. (i) $\begin{bmatrix} 5 & 3 \\ 3 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 17 & 5 & 14 \\ 38 & 11 & 32 \\ 59 & 24 & 50 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **Q.3.** $\begin{bmatrix} 43 \\ 44 \end{bmatrix}$ **Q.4.** x = -2 **Q.8.** $\begin{bmatrix} 16 \\ 16 \end{bmatrix}$

Q.10. $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$ **Q.13.** x = 1, y = -4, z = 3, u = -2 **Q.14.** $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$

Q.15. (i) Investment in Ist bond = Rs. 15,000; Investment in IInd bond = Rs.15,000
(ii) Investment in Ist bond = Rs. 5,000; Investment in IInd bond = Rs.15,000

Q.16. Ram Rs. 157.80; Mohinder Rs. 167.40 and Sham Rs. 281.40.

Q.17. Total amount Rs.1,597.20 **Q.18.** $x=1,2, y=3+3\sqrt{2}, 3-3\sqrt{2}$ **Q.19.**

$x=3, y=7, z=-2, w=14$ **Q.20.** 82 **Q.21.** $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$ **Q.22.** $x=-2, -14$

EXERCISE 3

Q.1. (a) $\begin{bmatrix} 1 & 6 \\ 2 & 8 \\ 4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 5 & 7 \\ 3 & -4 & 2 \\ 4 & -3 & 9 \end{bmatrix}$ **Q.4.** $\begin{bmatrix} -3 & 1 & 5 \\ -2 & 9 & 10 \end{bmatrix}$ **Q.5.** $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{2}}, z = \pm\frac{1}{\sqrt{3}}$

There is no substitute for hard work.
You must supplement reading by practising questions

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ADDRESS:
F-3/713, SANGAM VIHAR, NEW DELHI - 110062; PH.: **9810105034**

other chapters will be submitted soon