CBSE SUPPLEMENTARY EXAMINATION Set I, II, III

Max. Marks: 100 **Time Allowed: 3 Hours**

SECTION – A

(Question numbers 01 to 10 carry one mark each.)

Q01. If $f : \mathrm{R} \to \mathrm{R}$ is defined by $f(x) = 3x + 2$, then find $f(f(x))$.

Q02. If $\sin^{-1}\dfrac{1}{3} + \cos^{-1} x = \dfrac{\pi}{2}$, then find x.

Q03. If $\begin{pmatrix} a+b & 2 \\ 5 & b \end{pmatrix} = \begin{pmatrix} 6 & 5 \\ 2 & 2 \end{pmatrix}^{T}$, then find 'a'.

Q04. If A is a matrix of order 3×4 and B is a matrix of order 4×3, then find the order of matrix (AB).

Q05. If $A = \begin{pmatrix} 3 & 1 \\ 2 & -3 \end{pmatrix}$, then find $|adj\ A|$.

Q06. Evaluate: $\displaystyle\int \left(\frac{x^3 - 1}{x^2} \right) dx$.

Q07. Evaluate: $\displaystyle\int_{-\pi/4}^{\pi/4} \sin^3 x \, dx$.

Q08. Find a vector in the direction of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, which has a magnitude of 6 units.

Q09. Find the position vector of the mid-point of the line segment AB, where A is the point (3, 4,–2) and B is the point (1, 2, 4).

Q10. Find the distance of the point (2, 3, 4) from x-axis.

SECTION – B

(Question numbers 11 to 22 carry four marks each.)

Q11. Show that the function $f : \mathrm{R} \to \mathrm{R}$ given by $f(x) = ax + b$, where $a, b \in \mathrm{R}, a \neq 0$, is a bijection.

Q12. Prove that: $\cos^{-1}\dfrac{4}{5} + \cos^{-1}\dfrac{12}{13} = \cos^{-1}\dfrac{33}{65}$.

OR Solve for x: $\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}\left(\dfrac{8}{79}\right);\ x > 0$.

Q13. Using properties of determinants, prove that: $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$.

Q14. For what value of k is the function defined by $f(x) = \begin{cases} k(x^2+2), & \text{if } x \le 0 \\ 3x+1, & \text{if } x > 0 \end{cases}$ continuous at $x = 0$. Also write whether the function is continuous at $x = 1$.

Q15. If $y = (\cot^{-1} x)^2$, then show that $(x^2+1)^2 \dfrac{d^2y}{dx^2} + 2x(x^2+1)\dfrac{dy}{dx} = 2$.

OR If $x^y = e^{x-y}$, then prove that $\dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$.

Q16. Find the intervals in which the following function is (a) increasing (b) decreasing:

$$f(x) = 2x^3 - 9x^2 + 12x + 15.$$

OR Find the equation of tangent to the curve $y=\dfrac{x-7}{x^2-5x+6}$ at the point where it cuts the x-axis.

Q17. Evaluate: $\int\left[\log(\log x)+\dfrac{1}{(\log x)^2}\right]dx$. **OR** $\int\dfrac{dx}{(x^2+1)(x^2+2)}$.

Q18. Find the differential equation of the family of all circles touching the x-axis at the origin.

Q19. Solve the differential equation: $xy\log\left(\dfrac{y}{x}\right)dx+\left(y^2-x^2\log\left(\dfrac{y}{x}\right)\right)dy=0$.

Q20. Using vectors, find the area of the triangle with vertices A(2, 3, 5), B(3, 5, 8) and C(2, 7, 8).

Q21. Find the equation of the plane passing through the point A(1, 2, 1) and perpendicular to the line joining the points P(1, 4, 2) and Q(2, 3, 5). Also find the distance of this plane from the line $\dfrac{x+2}{2}=\dfrac{y-5}{-1}=\dfrac{z-7}{-1}$.

Q22. Find the probability distribution of the number of doublets in three throws of a pair of dice and hence find its mean.

SECTION – C

(Question numbers 23 to 29 carry six marks each.)

Q23. If $A=\begin{pmatrix}3 & 2 & 1\\ 4 & -1 & 2\\ 7 & 3 & -3\end{pmatrix}$, then find A^{-1}. Hence solve the following system of equations:

$3x+2y+z=6,\ 4x-y+2z=5,\ 7x+3y-3z=7$.

Q24. Find the maximum area of an isosceles triangle inscribed in the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$, with its vertex at one end of the major axis.

Q25. Using integration, find the area of the region $\{(x,y): x^2+y^2\le 16,\ x^2\le 6y\}$.

OR Find the area of the region $\{(x,y): x^2+y^2\le 1\le x+y\}$ by using integration.

Q26. Evaluate $\int_1^2 (x^2+5x)dx$ as limit of sums.

OR Evaluate: $\int[\sqrt{\tan x}+\sqrt{\cot x}]dx$.

Q27. Find the image of the point (1, 6, 3) in the line $\dfrac{x}{1}=\dfrac{y-1}{2}=\dfrac{z-2}{3}$. Also write the equation of the line joining the given point and its image and find the length of the segment joining the given point and its image.

Q28. A dealer deals in two items A and B. He has ₹15,000 to invest and a space to store at the most 80 pieces. Item A costs him ₹300 and item B costs him ₹150. He can sell items A and B at profits of ₹40 and ₹25 respectively. Assuming that he can sell all that he buys, formulate the above as a linear programming problem for maximum profit and solve it graphically.

Q29. In a bolt factory machines, A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from production and is found to be defective. What is the probability that it manufactured by the machine B?

☞ **NOTE:** Only those Questions from Set II and III are given here which are not in common with Set I.

SECTION – B (4 Marks)

Q01. Prove that: $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{36}{85}$.

Q02. Prove that: $\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{12}{13}=\sin^{-1}\frac{56}{65}$.

Q03. Solve for x: $\tan^{-1}\frac{x}{2}+\tan^{-1}\frac{x}{3}=\frac{\pi}{4};\ 0<x<\sqrt{6}$.

Q04. Solve for x: $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x)$.

Q05. Using properties of determinants, prove the following:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix}=(x-y)(y-z)(z-x)(x+y+z)$$

Q06. Using properties of determinants, prove that: $\begin{vmatrix} a+1 & 1 & 1 \\ 1 & b+1 & 1 \\ 1 & 1 & 1+c \end{vmatrix}=abc+ab+bc+ca$.

Q07. Find the value of a and b such that the function defined as follows is continuos:

$$f(x)=\begin{cases} x+2, & x\le 2 \\ ax+b, & 2<x<5 \\ 3x-2, & x\ge 5 \end{cases}$$

Q08. Find the value of k, for which the function f defined below is continuous:

$$f(x)=\begin{cases} \dfrac{k\cos x}{\pi-2x}, & x<\dfrac{\pi}{2} \\ 3, & x=\dfrac{\pi}{2} \\ \dfrac{3\tan 2x}{2x-\pi}, & x>\dfrac{\pi}{2} \end{cases}.$$

SECTION – C (6 Marks)

Q01. Find $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$. Use this to solve the system of equations:

$x-y+z=4,\ x-2y-2z=9,\ 2x+y+3z=1$.

Q02. If $A=\begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix}$, find A^{-1}. Hence solve the following system of equations:

$x+2y-3z=-4,\ 2x+3y+2z=2,\ 3x-3y-4z=11$.

Q03. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.

Q04. Prove that the radius of the right circular cylinder of greatest curved surface area, which can be inscribed in a given right circular cone, is half that of the cone.

ANSWERS OF CBSE SUPPLEMENTARY Exams. – Set I

Q01. $9x+8$ **Q02.** $\frac{1}{3}$ **Q03.** 4 **Q04.** 3×3 **Q05.** -11

Q06. $\frac{1}{x}+\frac{1}{2}x^2+k$ **Q07.** 0 **Q08.** $4\hat{i}-2\hat{j}+4\hat{k}$ **Q09.** $2\hat{i}+3\hat{j}+\hat{k}$

Q10. 5 **Q12.** **OR** $\frac{1}{4}$ **Q14.** $\text{k}=\frac{1}{2}$, Continuous at $x=1$

Q16. (a) $(-\infty,1)\cup(2,\infty)$ (b) $(1,2)$ **OR** $x-20y=7$

Q17. Put $\log_e x=t \Rightarrow x=e^t \Rightarrow x\left[\log\log x-\frac{1}{\log x}\right]+\text{k}$ **OR** $\tan^{-1}x-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\text{k}$

Q18. $\frac{dy}{dx}\left(y^2-x^2\right)+2xy=0$ **Q19.** $4y^2\log y=x^2\left[2\log\left(\frac{y}{x}\right)+1\right]$

Q20. $\frac{\sqrt{61}}{2}$ sq.units **Q21.** $x-y+3z=2,\ \frac{12}{11}\sqrt{11}$ units

Q22.

X	0	1	2	3
P(X)	125/216	75/216	15/216	1/216

, Mean $=\frac{1}{2}$

Q23. $\frac{1}{62}\begin{bmatrix}-3 & 9 & 52\\ 26 & -16 & -2\\ 19 & 5 & -11\end{bmatrix}$; $x=y=z=1$ **Q24.** $15\sqrt{3}$ sq.units

Q25. $\frac{16\pi+8\sqrt{3}}{3}$ sq.units **OR** $\frac{\pi-2}{4}$ sq.units **Q26.** $\frac{59}{6}$

OR There may be two different answers, depending upon your method:

$\sqrt{2}\sin^{-1}(\cos x-\sin x)+\text{k}$ or $\sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+\text{k}$

Q27. Image: (1,0,7); Eq. of Line: $\frac{x-1}{0}=\frac{y-6}{-3}=\frac{z-3}{2}$; $2\sqrt{13}$ units

Q28. To maximize: Z = ₹(40x + 25y), Subject to constraints: $x+y\le 80; 2x+y\le 10; x,y\ge 0$.

Also, Maximum value of Z = ₹2300 at (20, 60) **Q29.** $\frac{28}{69}$.

ANSWERS Of Questions from Set II & Set III which are not in Set I

Section – B

Q03. $x=1$ **Q04.** $x=\frac{\pi}{4},\frac{\pi}{2}$ **Q07.** $a=3, b=-2$ **Q08.** $\text{k}=6$.

Section – C

Q01. 81 ; $x=3, y=-2, z=-1$ **Q02.** $\frac{1}{67}\begin{bmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ -15 & 9 & -1\end{bmatrix}$; $x=3, y=-2, z=1$.