TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

GENERAL INSTRUCTIONS :-

CODE:- AG-13

1. All questions are compulsory.
2. The question paper consists of 34 questions divided into four sections A,B,C and D. Section – A comprises of 8 question of 1 mark each. Section – B comprises of 6 questions of 2 marks each. Section – C comprises of 10 questions of 3 marks each and Section – D comprises of 10 questions of 4 marks each.
3. Question numbers 1 to 8 in Sections – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four mark each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 6 printed pages.

MATHEMATICS	***CLASS X***	***(SA-1)***

Time : 3 to $3\frac{1}{4}$ Hours Maximum Marks : 90

SUMMATIVE ASSESSMENT –I (2013)

SECTION A

Q.1	The number $\frac{3-\sqrt{3}}{3+\sqrt{3}}$ is (A) rational (B) irrational (C) Both (D) Can't say Ans. B ; after rationalisation $2-\sqrt{3}$

Q.2	If $\sin 3\theta = \cos(\theta - 6^0)$, where (3θ) and $(\theta - 6^0)$ are both acute angles, then the value of θ is (A) 18^0 (B) 24^0 (C) 36^0 (D) 30^0 Ans. B : $90-3\theta=\theta-6 \Rightarrow 4\theta=96; \theta=24$
Q.3	$x^3 + 2x^2 + ax + b$ is exactly divisible by ($x^2 - 1$) .Then the value of 'a' and 'b' are (A) a = -1, b = -2 (B)a = 1, b = 2(C) a = -1, b = 2(D) a = 1,b = -2 Ans. a : a = -1 b = -2
Q.4	The median of the scores 13,23,12,18,26,19,14,25,11 is (A) 14 (B) 18 (C) 19 (D) 23 Ans. B :
Q.5	If θ is acute and $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3, then\ \theta =$ (A) 60^0 (B) 30^0 (C) 90^0 (D) 45^0 Ans. A :
Q.6	For what value of p does the system of equation $2x - py = 0, 3x + 4y = 0$ has non zero solution ? (A) p = - 6 (B) $p = -\frac{8}{3}$ (C) $p = -\frac{2}{3}$ (D) $p = -\frac{4}{5}$ Ans. B :
Q.7	$\Delta ABC \sim \Delta PQR$. If AB = 6cm, BC = 4 cm ,AC= 8cm, PR = 6cm, then PQ + QR = (A) 8cm (B) 10cm (C) 7.5 cm (D) 9 cm Ans. C :
Q.8	If $x = 2\sin^2\theta, y = 2\cos^2\theta + 1$ then the value of $x + y$ is (A) 2 (B) 3 (C) $\frac{1}{2}$ (d) 1 Ans. b :
	SECTION B
Q.9	If one zero of polynomial $3x^2 = 8x + 2k + 1$ is seven times the

other, then find the zeroes and value of k. Ans. $k=-\frac{5}{3}$

OR

Find all the zeros of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if two of its zeros are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$. **Sol.** Since $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$ are zeros of f(x). Therefore,

$\left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right)=\left(x^2-\frac{3}{2}\right)=\frac{2x^2-3}{2}$ or $2x^2 - 3$ is a factor of f(x).

$$
\begin{array}{r|l}
 & x^2 - x - 2 \\
2x^2-3 & 2x^4 - 2x^3 - 7x^2 + 3x + 6 \\
 & \underline{2x^4 \qquad\quad \mp 3x^2} \\
 & -2x^3 - 4x^2 + 3x + 6 \\
 & \underline{\mp 2x^3 \qquad \mp 3x} \\
 & -4x^2 + 6 \\
 & -4x^2 + 6 \\
 & \underline{+ \qquad -} \\
 & 0
\end{array}
$$

$\therefore$ $2x^4 - 2x^3 - 7x^2 + 3x + 6 = (2x^2 - 3)(x^2 - x - 2) = (2x^2 - 3)(x - 2)(x + 1)$

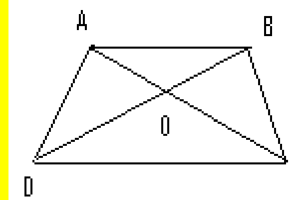
$=2\left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right)(x-2)(x+1)$ So, the zeros are $-\sqrt{\frac{3}{2}},\sqrt{\frac{3}{2}},2,-1$

Q.10 Find the LCM of $2^3 \times 3^2 \times 5 \times 11$, $2^4 \times 3^4 \times 5^2 \times 7$, and $2^5 \times 3^3 \times 5^3 \times 7^2 \times 11$. Ans. $L.C.M.=2^5\times3^4\times5^3\times7^2\times11$

Q.11 Find the mode of the following distribution of marks obtained by 80 students : Ans. mode = 36.92

Marks obtained	0-10	10-20	20-30	30-40	40-50
Number of students	5	11	12	30	22

Q.12 In the given figure $\frac{AO}{OC}=\frac{BO}{OD}=\frac{1}{2}$ and AB = 4cm , find the value of DC. **Ans. DC = 8 cm**



Q.13 Solve for x and y : $ax - by = a^2 - b^2, \frac{x}{a}+\frac{y}{b}=2$.

Ans. x = a ; y = b

Q.14 Evaluate : $\cos(40^0+\theta)-\sin(50^0-\theta)+\frac{\cos^2 40^0+\cos^2 50^0}{\sin^2 40^0+\sin^2 50^0}$.

Ans. 1

SECTION C

Q.15 Prove that sum of the squares of the diagonals of a parallelogram is equal

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Ph. :2337615; 4010685®, 2630601(O) Mobile : 9425109601; 9425110860;9425772164(P)

to sum of the squares of its sides.

OR

In a triangle ABC, D is the mid-point of BC and AE ⊥ BC. Prove that : $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$

Q.16 In a morning walk three persons step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that they can cover the distance in complete steps ?

Ans.LCM of 80 cm , 85 cm , 90 cm ie $12240cm = 122m40cm$

OR

Show that cube of any positive integer is of the form $4m$ or 4m + 1 or 4m + 3 where m is a positive integer.

Q.17 Prove that: $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \cos ecA$.

Q.18 Ritu can row downstream 20 km in 2 hrs. and upstream 4 km in 2 hrs. Find the speed of rowing in still water and the speed of the current .

Ans. still water = 6 km/hr speed of current = 4 km/hr

OR

In a competitive examination, one mark is awarded for each correct answer while ½ mark is deducted for each wrong answer. Sheela answered 120 question and got 90 marks. How many question did she answer correctly? Ans. 100

Q.19 Mean of the following data is 21.5.Find the missing value 'k'. Ans. K = 5

X	5	15	25	35	45
f	6	4	3	k	2

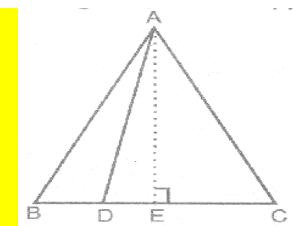
Q.20 The HCF & LCM of two numbers are 33 & 264 respectively. When the first number is divided by 2 the quotient is 33. Find the second number.

Q.21 Find the median of the following data : 5 , 17 , 23 , 14 , 29 , 11 , 43 , 13 , 53 , 36 . If 13 , 23 is replace by 72 , 49 . what will be the new median .

ans; median = 20 and new median = 65/2 ie = 32 . 5

Q.22 In $\Delta ABC, AD \perp BC$ and BD = $\frac{1}{3}CD$. Prove that $2CA^2 = 2AB^2 + BC^2$.

Q.23 In an equilateral triangle ABC, the side BC is trisected at D. Prove that 9 $AD^2 = 7AB^2$ **Sol.** ABC be can equilateral triangle and D be point on



BC such that BC = $\frac{1}{3}$BC (Given) Draw AE⊥BC, Join AD.BE = EC (Altitude drown from any vertex of an equilateral triangle bisects the opposite side)So, BE = EC = $\frac{BC}{2}$ In Δ ABC$AB^2 = AE^2 + EB^2$....(i)$AD^2 = AE^2 + ED^2$....(ii)From (i) and (ii)$AB^2 = AD^2 - ED^2 + EB^2$.$AB^2 = AD^2 - \frac{BC^2}{36} + \frac{BC^2}{4}$ (∴ BD + DE $= \frac{BC}{2} \Rightarrow \frac{BC}{3} + DE = \frac{BC}{2} \Rightarrow DE = \frac{BC}{6}$) $AB^2 + \frac{BC^2}{36} - \frac{BC^2}{4} = AD^2$ $(\therefore EB = \frac{BC}{2})$ $AB^2 + \frac{AB^2}{36} - \frac{AB^2}{4} = AD^2$

$(\therefore AB = BC)$ $\frac{36AB^2 + AB^2 - 9AB^2}{36} = AD^2 \Rightarrow \frac{28AB^2}{36} = AD^2$

$7AB^2 = 9AD^2$

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Ph. :2337615; 4010685®, 2630601(O) Mobile : 9425109601; 9425110860;9425772164(P)

Q.24	If one zero of the polynomial $p(x)=\left(k^2+4\right)x^2+13x+4k$ is reciprocal of other, then prove that k = 2 .
	SECTION D
Q.25	The mean of the following frequency table is 53.But the frequencies f_1 and f_2 in the classes 20-40 and 60-80 are missing. Find the missing frequencies . Ans. $f_1=18\&f_2=29$

Age (in years)	0-20	20-40	40-60	60-80	80-100	Total
Number of people	15	f_1	21	f_2	17	100

Q.26 Draw the graphs of the equations 4x – y = 4 & 4x + y = 12 .Deter mine the vertices of the triangle formed by the lines representing these equations and the x-axils. Shade the triangular region so formed. Also find its area

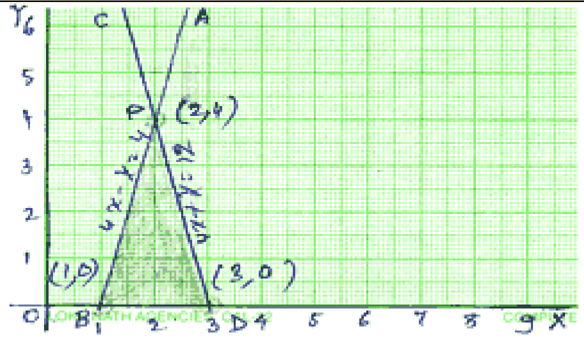
.**Solution: -** Let us take the equation $4x-y=4$

X	O	1	2
Y	-4	0	4

= We plot the points (0, -4), (1, 0) and (2, 4) on the graph paper and join them. We get a straight line. Now we take the line AB. 4x + y = 121

x	2	3	4
Y	4	0	-4

We plot the points (2, 4) (3, 0) and (4, -4) on the same graph paper. on joining them we get a line CD which intersect previous line AB. at P (2, 4)



AB intersects the x-axis at (1, 0) and CD intersects the x-axis at (3, 0) Hence the vertices of the triangle PBD are (2, 4), (1, 0) and (3, 0) . The required region is shaded.Area = $\frac{1}{2}\times 2\times 4=4$ sq unit .

Q.27 Show that: $\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta$.

OR

Show that : $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$.

Q.28 Prove that $(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2=7+\tan^2\theta+\cot^2\theta$.

Q.29 Determine the value of k so that the following linear equations have no solution: $(3k+1)x+3y-2=0$ & $\left(k^2+1\right)x+(k-2)y-5=0$.**Solution:-**

$\frac{a_1}{a_2}=\frac{3k+1}{k^2+1},\frac{b_1}{b_2}=\frac{3}{k-2},\frac{c_1}{c_2}=\frac{2}{5}$ For no solution, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ OR

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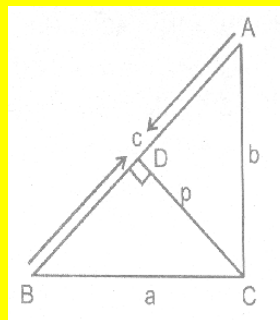
$$\frac{3k+1}{k^2+1} = \frac{3}{k-2} \neq \frac{2}{5} \quad Now, \ \frac{3k+1}{k^2+1} = \frac{3}{k-2}$$

Or, $(k - 2)(3k + 2) = 3(k^2 + 1)$ Or, $3k^2 - 5k - 2 = 3k^2 + 3$ Or, $-5k = 5$ Or, $k = -1$

Q.30 ABC is a right triangle, right-angled at C. Let BC = a, CA b, AB = c and let p be the length of perpendicular Form C on AB, prove that (i) cp = a b(ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.**Sol.** Let $CD \perp AB$. Then CD = p



$\therefore$ Area of $\Delta ABC = \frac{1}{2}$ (Base × height) = $\frac{1}{2}$ (AB × CD) $= \frac{1}{2}cp$ Also, Area of Δ ABC = $\frac{1}{2}$(BC × AC) = $\frac{1}{2}ab$

$\therefore \quad \frac{1}{2}cp = \frac{1}{2}ab$

$\Rightarrow$ CP = AB.

(ii) Since Δ ABC is a right triangle, right angled at C.

$\therefore \quad AB^2 = BC^2 + AC^2$

$$\Rightarrow \quad c^2 = a.. + b^2 \Rightarrow \left(\frac{ab}{p}\right)^2 = a^2 + b^2 \left[\because cp = ab \Rightarrow c = \frac{ab}{p}\right]$$

$$\Rightarrow \frac{a^2b^2}{p^2} = a^2 + b^2 \Rightarrow \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2} \Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Q.31 If $2\cos\theta - \sin\theta = x$ & $\cos\theta - 3\sin\theta = y$. prove that $2x^2 + y^2 - 2xy = 5$ ANS:

$$2\cos\theta - \sin\theta = x$$
$$\cos\theta - 3\sin\theta = y$$
$$(x-y)^2 = x^2 - 2xy + y^2$$
$$x^2 - 2xy + y^2 = (\cos\theta + 2\sin\theta)^2$$
$$= \cos^2\theta + 4\sin^2\theta + 4\cos\theta\sin\theta$$
$$= 1 + 3\sin^2\theta + 4\cos\theta\sin\theta$$

Adding x^2 on both sides

$$x^2 - 2xy + y^2 + x^2 = 1 + 3\sin^2\theta + 4\cos\theta\sin\theta + (2\cos\theta - \sin\theta)^2$$
$$2x^2 - 2xy + y^2 = 1 + 3\sin^2\theta + 4\cos\theta\sin\theta + 4\cos^2\theta + \sin^2\theta - 4\cos\theta\sin\theta$$
$$= 4\sin^2\theta + 4\cos^2\theta + 1$$
$$= 5$$
$$= RHS$$

Q.32 Show that one and only one out of n , n + 3 , n + 6 , n + 9 is divisible by 4 , where n is any positive integer .

OR

Prove that the product of three consecutive positive integer is divisible by 6 .

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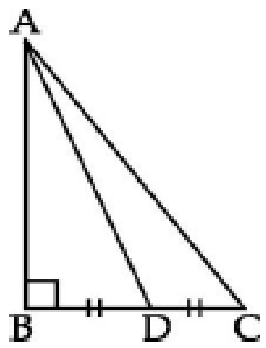
Q.33 The distribution below gives the weights of 30 students of a class. Find the mean and the median weight of the students. Ans mean = 57.1 ; median 56.67

C– I	40-45	45-50	50-55	55-60	60-65	65-70	70-75
F	2	3	8	6	6	3	2

Q.34 In right-angled triangle ABC in which $\angle B = 90^o$, if D is the mid-point of BC, prove that $AC^2 = 4AD^2 - 3AB^2$. ANS:



In right ΔABC

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = AB^2 + 4BD^2 \qquad [BC = 2\,BD]$$

$$AC^2 = AB^2 + 4\,[AD^2 - AB^2] \qquad [\because AD^2 = AB^2 + BD^2]$$

$$AC^2 = AB^2 + 4AD^2 - 4AB^2$$

$$AC^2 = 4AD^2 - 3AB^2$$

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HAPPINESS IS NOTHING MORE THAN GOOD HEALTH AND A BAD MEMORY.

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