Series : PTS/2

Code No. 13/9/2

Roll No. | 2 | 1 | 3 | 1 | 2 | | |

Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 02

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Time Allowed: 180 Minutes **Max. Marks: 100**

SECTION A

Q01. Let $A = \begin{pmatrix} d_1 & d_2 & d_3 & \ldots & d_n \end{pmatrix}$ be a diagonal matrix. What is the value of det.(A)?

Q02. If $A = \begin{bmatrix} 2 & 3 \\ k & 2 \end{bmatrix}$ and $A.\,\text{adj}A = 12\,I$ then, find the value of k.

Q03. Under what condition (A – B) (A + B) is equal to $A^2 + B^2$ such that orders of A and B are same?

Q04. Let $f : R \to R$ be defined by $f(x) = (3 - x^3)^{1/3}$. Determine $fof\,(x)$.

Q05. If the plane $4x + 4y - \lambda z = 0$ contains the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$, find the value of λ.

Q06. Evaluate the integral of $\int \frac{\sqrt{5+x^{10}}}{x^{16}}dx$.

Q07. Evaluate : $\sin\cos^{-1}(1) + \cos\sin^{-1}(1)$.

Q08. Find the value of m if the lines $\frac{x+3}{3} = \frac{y-1}{5m} = \frac{z+4}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-4}{2}$ are perpendicular to each other.

Q09. Find the unit vector in the direction of sum of vectors $\hat{i} - \hat{j}$ and $\hat{k} + \hat{j}$.

Q10. Find the integrating factor for the linear differential equation : $(y^2 - 1) + 2xy\frac{dy}{dx} = \left(\frac{2}{y^2-1}\right)\frac{dy}{dx}$.

SECTION B

Q11. Let $f(x) = \begin{cases} x + \sqrt{2}a, & 0 \le x \le \pi/4 \\ 2x\cot x + b, & \pi/4 < x \le \pi/2 \\ a\cos 2x - b\sin x, & \pi/2 < x \le \pi \end{cases}$ is a continuous function on $0 \le x \le \pi$. Then, determine the values of 'a' and 'b'.

What are your views about "learning"? Is "learning" a continuous process?

Q12. Solve : $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$.

Q13. Evaluate : $\int_0^{\pi} \frac{x\sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x - \pi}dx$.

Q14. Let * be a binary operation on N defined by a*b = HCF of a and b. Is * commutative? Is * associative? Does there exist identity for this binary operation on N?

OR Let $f : R \to R$ be defined as $f(x) = 10x + 7$. Find a function $g : R \to R$ such that we have $gof = fog = I_R$.

Q15. Express $2\hat{i} - \hat{j} + 3\hat{k}$ as the sum of a vector parallel and perpendicular to $2\hat{i} + 4\hat{j} - 2\hat{k}$.

Q16. Evaluate : $\int \frac{x^4}{(x-1)(x^2-1)}dx$. **OR** Evaluate : $\int \frac{x^2 + \sin^2 x}{1 + x^2}\sec^2 x\,dx$.

Q17. Using properties of determinants, evaluate: $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$.

Q18. Form the differential equation of the family of circles in 2^{nd} quadrant and touching coordinate axes.

OR Form the differential equation of the family of curves given by $(a + bx)\, e^{y/x} = x$.

Q19. Find the equation of the plane parallel to x-axis and which contains the line of intersection of the palnes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0$.

Q20. If $xy\log(x+y)=1$ then, prove that $\frac{dy}{dx}=-\frac{y}{x}\left(\frac{x^2y+x+y}{xy^2+x+y}\right)$.

OR If $y=\sqrt{x^2+1}-\log\left(\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right)$ then, prove that $\frac{dy}{dx}=\frac{\sqrt{x^2+1}}{x}$.

Q21. If $\cos y = x \cos (a + y)$ then, prove that $\sin a\,(dy/dx)=\cos^2(a+y)$.

Q22. Two thirds of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting first class is 0.25 and that of a boy is getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject.

SECTION C

Q23. Two trainee carpenters A and B earn ₹150 and ₹200 per day respectively. A can make 6 frames and 4 stools per day while B can make 10 frames and 4 stools per day. How many days shall each work if it is desired to produce at least 60 frames and 32 stools at a minimum labour cost? Solve graphically.

Q24. Triangle AOB is made in first quadrant of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where OA = a and OB = b. Find the area enclosed between the chord AB ad arc AB of ellipse.

"Differentiate the wastage of seconds; integrate the number of hours in a day." Comment.

Q25. Find the distance of the point (2, 3, 4) from the plane $3x+2y+2z+5=0$ measured parallel to the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$.

Q26. Evaluate : $\int\frac{\sqrt{1+x^2}}{1-x^2}dx$.

Q27. A farmer wants to construct a circular well and a square garden in his field. He wants to keep sum of their perimeters fixed. Then prove that the sum of their areas is least when the side of square garden is double the radius of the circular well. Do you think good planning can save energy, time and money?

OR Find the condition for the curves $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $xy=c^2$ to intersect orthogonally.

Q28. Let X denote the number of colleges where you will apply after your results and P(X = x) denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x)=\begin{cases} kx, \text{ if } x=0, \text{ or } 1 \\ 2kx, \text{ if } x=2 \\ k(5-x), \text{ if } x=3 \text{ or } 4\end{cases}, \text{ k is a positive constant.}$$

Find the mean and variance of the probability distribution.

Q29. For keeping fit, X people believe in morning walk, Y people believe in yoga and Z people join gym. Total no. of people are 70. Further 20%, 30% and 40% people are suffering from any diseases who believe in morning walk, yoga and gym respectively. Total number of such people is 21. If morning walk costs ₹0, yoga costs ₹500/month and gym costs ₹400/month and total expenditure is ₹23000.

(i) Formulate a matrix problem. (ii) Calculate the number of each type of people.

(iii) Why exercise is important for health?

OR Find the inverse of $\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$ using elementary transformations.

HINTS & ANSWERS *for* PTS XII – 02 [2013 - 2014]

Q01. $|A| = d_1 d_2 d_3 \ldots d_n$ **Q02.** $k = -3$ **Q03.** $AB = BA$ **Q04.** $fof(x) = x$

Q05. $\lambda = 5$ **Q06.** $-\frac{1}{75}\left(\frac{5}{x^{10}}+1\right)^{3/2}+k$ **Q08.** $m = -11/5$

Q07. 0 **Q09.** $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$ **Q10.** I.F. $= (y^2-1)$

Q11. $a = \frac{\pi}{2(1+2\sqrt{2})}, b = -\frac{\pi}{4(1+2\sqrt{2})}$. **Yes, 'learning' is a continuous process. A person learns at every moment of life from the daily activities happening around him.** **Q12.** $x = 13$

Q13. $\frac{8}{\pi^2}$ **Q14.** See NCERT Solutions Chapter 01 Ex. 1.4 Q. No. 8, visit **www.theOPGupta.WordPress.com** **OR** $g(x) = \frac{x-7}{10}$ **Q15.** $\frac{1}{2}(\hat{k}-\hat{i}-2\hat{j})+\frac{5}{2}(\hat{i}+\hat{j})$

Q16. $\frac{x^2}{2}+2\log|x-1|+\frac{1}{4}\log\left|\frac{x+1}{x-1}\right|-\frac{1}{2(x-1)}+k$ **OR** $\tan x - \tan^{-1}x + k$ **Q17.** $-x^3$

Q18. $(x+y)^2[(y')^2+1] = (x+yy')^2$ **OR** $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx}-y\right)^2$ **Q19.** $\vec{r}.(\hat{j}-3\hat{k})+6=0$ **Q22.** 0.27

Q23. To minimize : $Z = ₹(150x+200y)$; Subject to constraints : $6x+10y \geq 60$, $4x+4y \geq 32$; $x, y \geq 0$.

Minimum value of Z = ₹1350 at (5, 3). **Q24.** $\left(\frac{\pi-2}{4}\right)ab$ Sq.Units **Q25.** 7 Units

Q26. Let $I = \int\frac{\sqrt{1+x^2}}{1-x^2}dx = \int\frac{1+x^2}{(1-x^2)\sqrt{1+x^2}}dx = -\int\frac{-1-x^2}{(1-x^2)\sqrt{1+x^2}}dx = -\int\frac{1-x^2-2}{(1-x^2)\sqrt{1+x^2}}dx$

$\Rightarrow I = -\int\frac{1-x^2}{(1-x^2)\sqrt{1+x^2}}dx + \int\frac{2}{(1-x^2)\sqrt{1+x^2}}dx = -\int\frac{1}{\sqrt{1+x^2}}dx + I_2$.

Now put $x = \frac{1}{t}$ in $I_2 \Rightarrow dx = -\frac{1}{t^2}$. So $I_2 = -\int\frac{2t}{(t^2-1)\sqrt{1+t^2}}dt$.

Now put $t^2+1 = y^2 \Rightarrow 2t\,dt = 2y\,dy$.

We have, $I_2 = -\int\frac{2y}{(y^2-2)y}dy = -2\int\frac{1}{y^2-(\sqrt{2})^2}dy = -2\times\frac{1}{2\sqrt{2}}\log\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right| = \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{t^2+1}+\sqrt{2}}{\sqrt{t^2+1}-\sqrt{2}}\right|$

i.e., $I_2 = \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{x^2+1}+x\sqrt{2}}{\sqrt{x^2+1}-x\sqrt{2}}\right|$. So, $I = -\log\left|x+\sqrt{1+x^2}\right| + \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{x^2+1}+x\sqrt{2}}{\sqrt{x^2+1}-x\sqrt{2}}\right| + k$

Q27. **Yes, every work done in a planned way proves to be more fruitful. If a student makes a planning for his studies he can do wonders.** **OR** $a^2 = b^2$ **Q28.** 19/8, 47/64

Q29. (i) $x + y + z = 70$, $2x + 3y + 4z = 210$, $5y + 4z = 230$ (ii) $x = 20$, $y = 30$, $z = 20$ (iii) Exercise keeps fit and healthy to a person. **OR** $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$.

Good Luck & God Bless You!!!