Series : PTS/5

Code No. 13/10/3

Roll No. | 2 | 1 | 3 | 1 | 2 | | |

Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 05

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Time Allowed: 180 Minutes **Max. Marks: 100**

SECTION – A

Q01. If $f : [0, 1] \to [0, 1]$ and and $g : [0, 1] \to [0, 1]$ be the functions defined by $f(x) = x^2$ & $g(x) = 1 - x$, then find fog.

Q02. Find a skew symmetric matrix using A and A^T where $A = \begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}$.

Q03. Without expanding, evaluate : $\begin{vmatrix} 109 & 102 & 95 \\ 6 & 13 & 20 \\ 1 & -6 & -13 \end{vmatrix}$. **Q04.** Find the domain of the function $f(x) = \log x^2$.

Q05. Evaluate : $\int 3\sqrt{x}(1+\sqrt{x^3})\,dx$. **Q06.** Evaluate : $\int_0^{400\pi} \sqrt{1-\cos 2x}\,dx$.

Q07. Total cost C(x) associated with the provision of free mid-day meals to x students of a school in primary classes is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 50$. If the marginal contentment is given by rate of change $\frac{dC}{dx}$ of total cost, then write the marginal cost of food for 300 students. What value is shown here?

Q08. If $\overrightarrow{OP} = 3\vec{a} - 2\vec{b}$ and $\overrightarrow{OQ} = \vec{a} + \vec{b}$, find the position vector $\overrightarrow{OR}$ of a point R which divides the join of the points P and Q in the ratio of 2:1 internally. **Q09.** For what value of x, $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector?

Q10. Show that the points $(2,3,4), (-1,-2,1)$ and $(5,8,7)$ are collinear.

SECTION – B

Q11. Show that : $\tan^{-1}\left(\frac{1-x}{1+x}\right) - \tan^{-1}\left(\frac{1-y}{1+y}\right) = \sin^{-1}\left(\frac{y-x}{\sqrt{1+x^2}\sqrt{1+y^2}}\right)$.

OR Solve for x : $\tan^{-1}\sqrt{x^2+x} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$.

Q12. Discuss the commutativity and associativity of binary operation * defined on Q by the rule given as : $a * b = a - b + ab$ for all $a, b \in R$.

Q13. Find the equation of a plane which passes through (–1, 3, 2) and is perpendicular to each of the planes given as x + 2y + 3z = 5 and 3x + 3y + z = 0.

Q14. Discuss the differentiability of $f(x) = x|x|$ at $x = 0$.

OR Let $f(x) = \sin x$, $g(x) = x^2$ and $h(x) = \log_e x$. If V (x) = *hofog* (x), then prove that :

$$\frac{d^2V}{dx^2} = 2\cot x^2 - 4x^2 \operatorname{cosec}^2 x^2.$$

Q15. If $x = a\left\{\cos\theta + \log\tan\left(\frac{\theta}{2}\right)\right\}$ and $y = a\sin\theta$ then, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

Q16. Discuss the continuity of $f(x)$ at $x = 0$, where $f(x)=\begin{cases}\dfrac{\cos ax-\cos bx}{x^2}, \text{ when } x\neq 0\\ \dfrac{b^2-a^2}{2}, \text{ when } x=0\end{cases}$?

Q17. Evaluate : $\int \dfrac{\sin x}{\sin 4x}dx$. **OR** Evaluate : $\int \dfrac{\cot x+\cot^3 x}{1+\cot^3 x}dx$.

Q18. Evaluate : $\int \sqrt{\dfrac{\sin(x-\alpha)}{\sin(x+\alpha)}}\,dx$. **Q19.** Evaluate : $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x)\,dx$.

Q20. Using properties of determinants, prove that : $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$.

Q21. If $\vec{a},\vec{b},\vec{c}$ are three mutually perpendicular vectors of equal magnitude, prove that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined with vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

OR If $\vec{a},\vec{b},\vec{c}$ are non-coplanar vectors and $\vec{p}=\dfrac{\vec{b}\times\vec{c}}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]}$, $\vec{q}=\dfrac{\vec{c}\times\vec{a}}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]}$ and $\vec{r}=\dfrac{\vec{a}\times\vec{b}}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]}$ then, show that $\vec{p}.(\vec{a}+\vec{b})+\vec{q}.(\vec{b}+\vec{c})+\vec{r}.(\vec{c}+\vec{a})=3$.

Q22. Out of a group of 30 honest people, 20 always speak the truth. Two persons are selected at random from the group. Find the probability distribution of the number of selected persons who speak the truth. Also find the mean of the distribution. What values are described in this question?

SECTION – C

Q23. Using matrix method, solve the following system of equations :
$x + 2y + z = 7$, $x – y + z = 4$, $x + 3y + 2z = 10$.
Suppose x represents the number of persons who take food at home, y represents the number of persons who take junk food in market and z represent the number of persons who take food at hotel. Which way of taking food you prefer and why?

OR In a survey of 20 richest person of three cities A, B and C it is found that in city A, 5 believe in honesty, 10 in hardwork, 5 in unfair means while in city B, 5 believe in honesty, 8 in hardwork, 7 in unfair means and in city C, 6 believe in honesty, 8 in hardwork, 6 in unfair means. If the per day income of these richest persons of cities A, B and C are ₹32.5K, ₹30.5K and ₹31K respectively, then find the per day income of each type of person by matrix method. (Here ₹1K means ₹1000.)
(i) Which type of persons have more per day income?
(ii) According to you, which type of person is better for country?

Q24. A water tank with rectangular base and sides, open at the top is to be constructed so that its depth is 2m and volume is $8m^3$. If building of tank costs ₹70 per m^2 for the base and ₹45 per m^2 for the sides, what is the cost of least expensive tank? What is the importance of 'save water' movement?

Q25. Using integrals, find the area of the triangular region the equations of whose sides are given as follow :
$y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

Q26. Find the particular solution of : $(3xy+y^2)dx+(x^2+xy)dy=0$; given that for x = 1, y = 1.

OR Show that $x\cos\left(\dfrac{y}{x}\right)\dfrac{dy}{dx}=y\cos\left(\dfrac{y}{x}\right)+x$ is a homogeneous differential equation. Hence solve it.

Q27. Show that the lines $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=4\hat{i}-\hat{k}+\mu(2\hat{i}+3\hat{k})$ intersect. Hence find their point of intersection and, the equation of plane containing them.

Q28. An aeroplane can carry a maximum of 200 passengers. A profit of ₹1000 is made on each executive class ticket and a profit of ₹600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

Q29. In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probability of getting a special chest diseases are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the diseases. What is the probability that the selected person is a smoker and non-vegetarian? What value is reflected in this question?

HINTS & ANSWERS *for* PTS XII – 05 [2013 - 2014]

Q01. $(1-x)^2$ **Q02.** $\begin{bmatrix} 0 & 8 \\ -8 & 0 \end{bmatrix}$

Q03. Apply $R_1 \to R_1 - R_3 \Rightarrow R_2 \to R_2 + R_3 \Rightarrow R_1$ and R_3 are proportional. So $\Delta = 0$.

Q04. $R - \{0\}$ **Q05.** $2x\sqrt{x} + x^3 + C$ **Q06.** 0 **Q07.** ₹1368 **Q08.** $\frac{5\vec{a}}{3}$

Q09. $\pm\frac{1}{\sqrt{3}}$ **Q11.** **OR** $x = 0, -1$

Q12. Neither commutative nor associative **Q13.** $7x - 8y + 3z + 25 = 0$

Q14. Function $f(x)$ is differentiable at x = 0 **OR** Obtain V (x) = $\log_e \sin(x^2)$ and then proceed.

Q15. $\frac{8}{9a}$ **Q16.** Function $f(x)$ is continuous at x = 0

Q17. $\frac{1}{4\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| - \frac{1}{8}\log\left|\frac{1+\sin x}{1-\sin x}\right| + C$

OR $\frac{1}{3}\log|\cot x + 1| - \frac{1}{6}\log|\cot^2 x - \cot x + 1| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\cot x - 1}{\sqrt{3}}\right) + C$

Q18. $-\cos\alpha \sin^{-1}\left(\frac{\cos x}{\sin\alpha}\right) - \sin\alpha \log|\sin x + \sqrt{\sin^2 x - \sin^2\alpha}| + C$ **Q19.** $\frac{\pi}{2} - 1$

Q21. See Hints/Answers in OPG Vol. 2

Q22. **Probability Distribution :**

X	0	1	2
P(X)	90/870	400/870	380/870

Values described here : Honesty, Truthfullness.

Q23. $A^{-1} = \frac{1}{-3}\begin{bmatrix} -5 & -1 & 3 \\ -1 & 1 & 0 \\ 4 & -1 & -3 \end{bmatrix}$; x = 3, y = 1, z = 2 **OR** Per day income of person who believe in honesty = ₹1500, Per day income of person who believe in hardwork = ₹2000 and, Per day income of person who believe in unfair means = ₹1000. (i) The persons who believe in hardwork has more per day income. (ii) A person who believes in hardwork and honesty, is better for country.

Q24. C = ₹ [280 + 180 ($l + b$)], where $2lb = 8$. Minimum Cost = ₹1000.

Q25. 8sq.units **Q26.** $x^2y(y + 2x) = 3$ **OR** $\sin\left(\frac{y}{x}\right) = \log|x| + C$

Q27. Point : (4, 0, –1); $3x + 9y - 2z = 14$

Q28. Assume that the number of tickets to be sold for the executive class and economy class be respectively x and y.

To maximize: $Z = ₹(1000x + 600y)$

Subject to constraints: $x + y \le 200, x \ge 20, y \ge 4x, x \ge 0, y \ge 0$

Max.Z = ₹136000 at (40,160) **Q29.** $\frac{28}{45}$.

Good Luck & God Bless You!!!