Sample Paper – 2014
Class – XII
Subject – Mathematics

Time:-3hours Maximum Marks- 100

General Instructions:-

i) All questions are compulsory
ii) The question paper consists of 29 questions divided into three Sections A, B, C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 07 questions of six marks each.
iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
iv) There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions
v) Use of calculators is not permitted. You may ask for logarithmic tables, if required

Section- A

1) What is the principle value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

2) If $sin\left(sin^{-1}\frac{1}{2}+cos^{-1}x\right)$ = 1, find x

3) If A is non-singular square matrix such that A^{-1} = $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then find $(A^T)^{-1}$

4) Find the value of x, such that the points (0, 2), (1, x) , (3, 1) are collinear.

5) If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α is A an identity matrix ?

6) The side of an equilateral triangle is increasing at the rate of 0.5 cm/sec. Find the rate of increase of its perimeter.

7) Write the order and degree of the differential equation $\cos\left(\frac{dy}{dx}\right) = x + a$

8) Find the value of λ , if the vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ and $\vec{c} = \hat{\imath} - 3\hat{\jmath} + \lambda\hat{k}$ are co-planar.

9) If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}).(\vec{x} + \vec{a})$ = 8, find $|\vec{x}|$

10) Find the angle between the line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath} + 2\hat{k})$ and the plane $\vec{r}.(\hat{\imath} + \hat{\jmath} + \hat{k})$ = 3

Section- B

11) Given a non-empty set X, let * : P(X) × P(X) → P(X) be defined as A*B = (A- B)∪ (B – A), ∀ A, B ∈ P(X). Show that the empty set ∅ is the identity for the operation * and all the elements A of P(X) are invertible with A^{-1} = A

12) Prove that : $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \sqrt{\frac{1 + x^2}{2 + x^2}}$

(OR)

Solve for x : $\cos^{-1}\left(\frac{x^2 - 1}{x^2 + 1}\right) + \tan^{-1}\left(\frac{2x}{x^2 - 1}\right) = \frac{2\pi}{3}$

13) Using properties of determinants prove that

$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$$

14) If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2 y = 0$

15) If $f(x) = |x|^3$, show that $f''(x)$ exists for all real x and find it.

16) Discuss the continuity of the following function at $x = 0$.

$$f(x) = \begin{cases} \dfrac{x^4 + 2x^3 + x^2}{\tan^{-1} x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

(OR)

Show that the function 'f' defined by $f(x) = \begin{cases} 3x - 2, & 0 < x \le 1 \\ 2x^2 - x, & 1 < x \le 2 \\ 5x - 4, & x > 2 \end{cases}$ is continuous at $x = 2$, but not differentiable.

17) Evaluate: $\int \frac{1}{\sqrt{\sin^3 x \sin(x + \alpha)}} dx$

(OR)

Evaluate: $\int \frac{\sqrt{1 + x^2}[\log(x^2 + 1) - 2\log x]}{x^4} dx$

18) Evaluate: $\int \sqrt{\frac{1 - \sqrt{x}}{1 + \sqrt{x}}}\, dx$

19) Evaluate: $\int_0^{\pi} \frac{x\, dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

20) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$

21) Find the equation of the plane that contains the point (1, -1, 2) and is perpendicular to each of the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$

(OR)

Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$, $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

22) There is a group of 50 people who are patriotic out of which 20 believes in non violence. Two persons are selected at random out of them, write the probability distribution for the selected persons who are non violent. Also find the mean of the distribution. Explain the importance of Non violence in patriotism.

Section- C

23) Three values punctuality (x) , obedience(y) and honesty (z) are represented by the following system of equation: .

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$, where $x \neq 0, y \neq 0, z \neq 0$.
Solve the system of equation by matrix method .
Which value you prefer to be rewarded most and why?

24) Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one –third that of the cone and the greatest volume of cylinder is $\frac{4}{27} \pi h^3 \tan^2\alpha$

(OR)

Find the equation of tangents to the curve $y = \cos(x + y), -2\pi \leq x \leq 2\pi$ that are parallel to the line $x + 2y = 0$

25) Find the area of the region enclosed between the two circles: $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$

26) Show that the differential equation $2ye^{\frac{x}{y}}\,dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ is homogeneous and find its particular solution, given that, $x = 0$ when $y = 1$.

27) Find the distance of the point (-1, - 5, - 10) from the point of intersection of the line $\vec{r} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k} + \lambda\left(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{\imath} - \hat{\jmath} + \hat{k}\right) = 5$

28) There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q respectively 8 and 6 units. The cost of transportation per unit is given below:

From/ To	Cost (In Rs.)		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. What will be the minimum transportation cost? Write the importance to minimize the transportation cost.

29) A and B throw a die alternatively till one of them gets a ‘6’ and wins the game. Find their respective probabilities of winning, if A starts first.

(OR)

Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 blacks balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black

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