

INTEGRATION

(ADDITIONAL 6 MARKS SUMS FOR 20013-14)

1. Evaluate $\int_1^2 (x^2 + 5x) dx$ as limit of sum.

2. Evaluate $\int_1^3 (2x^2 + 5x) dx$ as limit of sum

3. Show that $\int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) dx = \sqrt{2}\pi$

4. Evaluate $\int_0^1 \cot^{-1}(1 - x + x^2) dx$.

5. Evaluate $\int (x^2 \cot^{-1} x) dx$

6. Evaluate $\int (x+3)\sqrt{3-4x-x^2} dx$

7. Evaluate $\int (6x+5)\sqrt{6+x-x^2} dx$

8. $\int \frac{x^2+4}{x^4+16} dx$

9. $\int \frac{1}{x^2} \sin^{-1} x dx$

10. $\int \log(x + \sqrt{x^2 + a^2}) dx$

11. $\int \left(\frac{1}{x^2} \tan^{-1} x \right) dx$

12. $\int_{-1}^{3/2} |x \sin \pi x| dx$

13. $\int \sqrt{\tan x} dx$

14. $\int (x^2 \tan^{-1} x) dx$

15. $\int (x^3 \tan^{-1} x) dx$

16. $\int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$

17. $\int_0^{\pi/2} \log \sin x dx$

18. $\int_0^{\pi} x \log \sin x dx$

19. $\int \frac{dx}{x^4+1}$

20. $\int \frac{x^2+1}{x^4+x^2+1} dx$

21. $\int \frac{\sec x \tan x + \sec^2 x}{(\sec x + \tan x)^n} dx$

22. $\int_0^{\pi/2} \frac{\sec^2 x}{(\sec x + \tan x)^n} dx$

23. $\int \frac{dx}{\sqrt{1+\sin x}}$

ANSWERS

$$4. \left(\text{Ans: } \frac{\pi}{2} - \log 2 \right)$$

$$5. \left(\text{Ans: } \frac{x^3}{3} \cot^{-1} x + \frac{1}{6} x^2 - \frac{1}{6} \log |1+x^2| + c \right)$$

$$6. \left(\text{Ans: } \frac{-1}{\sqrt{3-4x-x^2}} + \frac{x+2\sqrt{3-4x-x^2}}{2} + c \right)$$

$$7. \left(\text{Ans: } -2(6+x-x^2)^{3/2} + 2(2x-1)\sqrt{6+x-x^2} + 25 \sin^{-1} \frac{2x-1}{5} + c \right)$$

$$8. \left(\text{Ans: } \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x^2-4}{2\sqrt{2}x} \right) + c \right)$$

$$9. \left(\text{Ans: } \frac{-1}{x} \sin^{-1} x + \log \left| \frac{1-\sqrt{1-x^2}}{x} \right| + c \right)$$

$$10. \left(\text{Ans: } x \log |x + \sqrt{x^2+a^2}| - \sqrt{x^2+a^2} + c \right)$$

$$11. \left(\text{Ans: } -\frac{\tan^{-1} x}{x} + \log \left| \frac{x}{\sqrt{1+x^2}} \right| + c \right)$$

$$12. \left(\text{Ans: } \frac{3}{\pi} + \frac{1}{\pi^2} \right)$$

$$13. \left(\text{Ans: } \frac{1}{\sqrt{2}} \tan^{-1} \frac{\tan x - 1}{\sqrt{2} \tan x} + \frac{1}{2\sqrt{2}} \log \left| \frac{\tan x - \sqrt{2} \tan x + 1}{\tan x + \sqrt{2} \tan x + 1} \right| + c \right)$$

$$14. \left(\text{Ans: } \frac{x^3}{3} \tan^{-1} x - \frac{1}{6} x^2 + \frac{1}{6} \log(x^2+1) + c \right)$$

$$15. \left(\text{Ans: } \frac{x^4}{4} \tan^{-1} x - \frac{x^3}{12} + \frac{x}{4} - \frac{1}{4} \tan^{-1} x + c \right)$$

$$16. \left(\text{Ans: } \frac{x}{2} \cos^{-1} x - \frac{\sqrt{1-x^2}}{2} + c \right)$$

$$17. \left(\text{Ans: } -\frac{\pi}{2} \log 2 \right)$$

$$18. \left(\text{Ans: } -\pi^2 \log 2 \right)$$

$$19. \left(\text{Ans: } \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x^2-1}{\sqrt{2}x} \right) - \frac{1}{4\sqrt{2}} \log \left| \frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1} \right| + c \right)$$

$$20. \left(\text{Ans: } \tan^{-1} \left(x - \frac{1}{x} \right) + c \right)$$

$$21. \left(\text{Ans: } \frac{1}{n+1} (\sec x - \tan x)^{n+1} + c \right)$$

$$22. \left(\text{Ans: } \frac{n}{n^2-1} \right) \quad 23. \left(\text{Ans: } \sqrt{2} \log \tan \left(\frac{x}{4} + \frac{\pi}{8} \right) + c \right)$$