2013-14 - MATHEMATICS MODEL PAPER- 1

By D.Vasu Raj

SECTION – A(10 X 1 = 10 MARKS)

1. Let $*$ be a binary operation defined on Q. such that a$*$b = a+b-ab, where $a,b \in Q-\{1\}$. Then find $4*3$.

2. Find the value of $\cos\left(\cos^{-1}(\frac{-\sqrt{3}}{2})+\frac{\pi}{6}\right)$

3. If A is of order 3 x 4 and BA is of order 2 x4, then the order of B is - - - -

4. Find x and y if $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

5. If for a matrix A, $|\text{adjA}|=64$ where A is a 3rd order square matrix, then find $|3A|$

6. Find the projection of $7\vec{i}+\vec{j}-4\vec{k}$ *on* $2\vec{i}+6\vec{j}+3\vec{k}$.

7. *If* $\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right]=36$ then find the value of $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]$.

8. Find the direction cosines of the line passing through the points (1, 3,–2) and (2, –2, 3).

9. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\sin^5 x\right)dx$

10. Evaluate $\int\frac{x+2}{x^2+4x}dx$

SECTION – B(12 X 4 = 48 MARKS)

11. Show that the function f:R$\to$R defined by f(x) = $\frac{2x-1}{3}$, x$\in$ R is one-one(Injective) and Onto(Surjective). Also find the inverse of f.

12. Find the value of $\tan^{-1}\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right), |x|<1, y>0, xy<1$ (OR) Prove that $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$,

13. Show that $\begin{vmatrix}a+b+2c & a & b\\ c & b+c+2a & b\\ c & a & c+a+2b\end{vmatrix}=2(a+b+c)^3$

14. Find p so that $f(x) = \begin{cases} \dfrac{\sqrt{1+px}-\sqrt{1-px}}{x}; & -1 \le x < 0 \\ \dfrac{2x+1}{x-2}; & 0 \le x \le 1 \end{cases}$ is continuous at x = 0

15. If $y = e^{m\sin^{-1}x}$, show that $(1-x^2)y_2 - xy_1 - m^2y = 0$.

16. Find the intervals in which f(x) = sin3x is increasing or decreasing

(OR)

Find the equation of the tangent to the curve $y = \dfrac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

17. Evaluate $\int \dfrac{x^2+1}{x^2-5x+6}dx$ (OR) Evaluate $\int_0^{\frac{\pi}{2}} \dfrac{1}{1+\cot^3 x}dx$

18. Solve $\dfrac{dy}{dx} + y\cot x = 2x + x^2\cot x$ given that y(0) = 0.

19. Solve $(3xy+y^2)dx = (x^2+xy)dy$

20. Show that the vectors $\vec{a} = 3\vec{i} - 4\vec{j} - 4\vec{k}, \vec{b} = 2i - \vec{j} + \vec{k}$ *and* $\vec{c} = \vec{i} - 3\vec{j} - 5\vec{k}$, respectively form the vertices of a right angled triangle.

(OR)

If $\vec{p} = -3\hat{i} + 4\hat{j} - 7\hat{k}$ and $\vec{q} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ then find $\vec{p}x\vec{q}$. Verify that $\vec{p}$ *and* $\vec{p}x\vec{q}$ are perpendicular to each other.

21. Find the shortest distance between the lines $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$.

22. A clever student used a biased coin so that the head is 3 times as likely to occur as tail. If the coin is tossed twice find the probability distribution and mean of number of tails. Is this a good tendency? Justify your answer.

SECTION – B(7 X 6 = 42 MARKS)

23. A wire of length 28cm. is to be cut into two pieces. One of the pieces is to be made into a square and other into a circle. What could be the lengths of the two pieces so that the combined area of the square and the circle is minimum?

24. There are three families. First family consists of 2 male members, 4 female members and 3 children. Second family consists of 3 male members, 3 female members and 2 children. Third family consists of 2 male members, 2 female members and 5 children. Male member earns Rs 500 per day and spends Rs 300 per day. Female member earns Rs 400 per day and spends Rs 250 per day child member spends Rs 40 per day. Find the money each family saves per day using matrices? What is the necessity of saving in the family?

25. Two tailors A and B earn Rs.150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days

shall each work if it is desired to produce 60 shirts and 32 pants at a minimum labour cost. What do you prefer whether team work or individual work and why?

26. *Evaluate* $\int_0^{\pi} x \log \sin x \, dx$

27. A variable plane is at a constant distance 3p from the origin and meets the axes in A,B,C respectively, then show that locus of the centroid of triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$

(OR)

Find the ratio in which the plane x-2y+3z = 17 divides the line joining the points (-2,4,7) and (3,-5,8). Also obtain the co-ordinate of the point of intersection.

28. Draw a rough sketch of $\{(x,y): x^2+y^2 \le 4, x+y \ge 2\}$ .Find the area of the region enclosed.

29. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup

WISH YOU ALL THE BEST.

D.VASU RAJ
vraj626@gmail.com
Velammal knowledge park
Panchetty,
Chennai, TamilNadu