

OP Gupta's

Max. Marks: 50 [Ch 04 + 09] **Time Allowed: 90 Minutes**

Q01. If f is a function satisfying f(x + y) = f(x) f(y) for all $x, y \in \mathbb{N}$ such that f(1) = 3 and $\sum_{i=1}^{n} f(x) = 120$.

Determine the value of n.

- Q02. Let a, b, c and d are in GP such that a, b are roots of $x^2 3x + p = 0$ & c, d are roots of $x^2 12x + q = 0$. Prove that (q + p) : (q - p) = 17:15.
- **Q03.** Show that $\frac{1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + ... + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + 3^2 \times 4 + ... + n^2 \times (n+1)} = \frac{3n+5}{3n+1}$
- **Q04.** Find the sum of the series : 3 + 7 + 13 + 21 + 31 + ...

Evaluate: $1(1) + 2(1+2) + 3(1+2+3) + \dots$ up to n terms.

- Q05. The sum of two numbers is 6 times their geometric mean. Determine the ratio of these numbers.
- **Q06.** The ratio of sums of n terms of two arithmetic progressions is (3n + 4):(5n + 6). Find the ratio of their fifth terms. What is the ratio of their first terms?
- Q07. The sum of first three terms of a GP is 16 and the sum of next three terms is 128. Find the first term and the common ratio of GP.
- **Q08.** If one geometric mean G and two arithmetic means p and q be inserted between two positive numbers, show that $G^2 = (2p - q)(2q - p)$.
 - Show that the product of n geometric means between two given numbers is equal to the nth power of the single geometric mean between them.
- **Q09.** Let $x = 1 + a + a^2 + ...$ upto infinity, |a| < 1 and $y = 1 + b + b^2 + ...$ upto infinity, |b| < 1. Show that :

$$1 + ab + a^2b^2 + \dots upto infinity = \frac{xy}{x + y - 1}$$

- Q10. In an increasing GP, the sum of first and the last term is 66, the product of the second and the last but one term is 128. If the sum of the series is 126, find the number of terms of the series.
- Q11. Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an AP

- and the ratio of 7th and (m 1)th numbers is 5: 9. Find the value of m.

 Q12. Sum the series 3 + 3.3 + 3.33 + 3.333 + ... upto 30 terms.

 Q13. Using mathematical induction, show that "3²ⁿ⁺² 8n 9" is divisible by 8.

 Q14. If a, b, c, d and p are different real numbers such that (a² + b² + c²) p² 2(ab + bc + cd) p + (b² + c² + c²) d^2) ≤ 0 , then show that a, b, c and d are in G.P.
- Q15. (a) Let P(n) is a statement "12n + 5 is a multiple of 13". Show that P(2) is false whereas P(5) is true.
 - (b) Write the 45^{th} term of the series : $2^3 3^3 + 4^3 5^3 + 6^3 7^3 + \dots$ upto n terms.

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