TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

CODE:- AG-TS-4-8199 **REG.NO:-TMC -D/79/89/36**

General Instructions :-

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one If the alternatives in all such questions.
4. Use of calculator is not permitted.
5. Please check that this question paper contains 6 printed pages.
6. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

Time : 3 Hours Maximum Marks : 100

PRE-BOARD EXAMINATION 2013 -14

CLASS – XII ***CBSE*** ***MATHEMATICS***

PART – A

Q.1	If $\vec{a}$ & $\vec{b}$ are two unit vectors inclined to x–axis at angles 30^0 & 120^0 respectively, then write the value of $\left\|\vec{a}+\vec{b}\right\|$. **ANS :** $\sqrt{2}$

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Q.2	Write the value of $\int_0^{\pi/2} \log\left[\frac{3+5\cos x}{3+5\sin x}\right]dx$. I = 0
Q.3	For two non zero vectors $\vec{a}$ and $\vec{b}$ write when $\|\vec{a}+\vec{b}\| = \|\vec{a}\|+\|\vec{b}\|$ holds. Ans a & b are like parallel vector .
Q.4	A matrix A of order 3 × 3 has determinant 5. What s the value of |3A|? |3A| = 135
Q.5	Write the smallest equivalence relation R on Set A = {1, 2, 3} . ANS : R = {(1,1), (2, 2), (3, 3)}
Q.6	A four digit number is formed using the digits 1,2,3,5 with no repetitions. Find the probability that the numbers is divisible by 5. $\frac{6}{64}=\frac{1}{4}$
Q.7	Determine order and degree(if defined) of differential equation , $\left(\frac{d^4y}{dx^4}\right)+a\sin\left(\frac{d^3y}{dx^3}\right)=0$. Order is 4 and degree is not defined
Q.8	Evaluate , $\int_0^{1.5}[x]dx$. (where [x] is greatest integer function). 0.5
Q.9	If $4\sin^{-1}x+\cos^{-1}x=\pi$ then find the value of x . $x=\frac{1}{2}$
Q.10	Find a, for which f(x) = a(x + sin x) is increasing. a > 0

PART – B

Q.11	Evaluate : $\int \frac{2+\sin x}{1+\cos x}.e^{x/2}.dx$ $I=2\tan\frac{x}{2}\times e^{x/2}+c$ **OR**

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Evaluate : $\int \frac{5x}{(x+1)(x^2+1)}dx$. Ans :

$$I = \int \frac{5x}{(x+1)(x^2+9)}dx$$

$$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+c}{x^2+9} \Rightarrow A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{9}{2}$$

$$\Rightarrow I = -\frac{1}{2}\int \frac{1}{x+1}dx + \frac{1}{2}\int \frac{x+9}{x^2+9}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\frac{x}{3} + C$$

Q.12 A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi – vertical angle is $\tan^{-1}(1/2)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10m.

Rate of water level = $\frac{1}{5\pi} m/\min ute$

Q.13 If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove that $(aI + bA)^n = a^n . I + na^{n-1} bA$ where I is a unit matrix of order 2 and n is a positive integer.

OR

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If a, b and c are real numbers and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$. Show that either a + b + c = 0 or a = b = c.

Q.14 Show that the function $y = (A + Bx)e^{3x}$ is a solution of the equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0.$

Q.15 Find the shortest distance between the lines, whose equations are $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{10-z}{-7}$ and $\frac{x-15}{3} = \frac{58-2y}{-16} = \frac{z-5}{-5}$ .Also find the angle between two lines . $\theta = \cos^{-1}\frac{-154}{\sqrt{314}\times\sqrt{98}} = -\frac{11}{\sqrt{157}} Or \frac{11}{\sqrt{157}}$ & S.D. = 14

OR

Find the equation of the plane passing through the intersection of the planes, 2x + 3y – z + 1 = 0; x + y – 2z + 3 = 0 and perpendicular the plane 3x – y – 2z – 4 = 0. also the inclination of this plane with the xy- plane. $7x + 13y + 4z - 9 = 0, \theta = \cos^{-1}\frac{4}{\sqrt{234}}$

Q.16 Show that the differential equations $2y\, e^{x/y} dx + (y - 2x\, e^{x/y})dy = 0$ is homogeneous and find its particular solution given that x = 0 when y = 1. Ans : $\frac{dx}{dy} = \frac{2xe^{x/y} - y}{2ye^{x/y}}$ Sol of differential equation $2e^{x/y} + \log y = c \therefore 2e^{x/y} + \log y = 2$

OR

The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If

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the population of the village was 20000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009?

Answer : Let the population at any instant (t) be y. It is given that the rate of increase of population is proportional to the number of inhabitants at any instant.

$$\therefore \frac{dy}{dt} \propto y$$

$$\Rightarrow \frac{dy}{dt} = ky \qquad (k \text{ is a constant})$$

$$\Rightarrow \frac{dy}{y} = kdt$$

Integrating both sides, we get:

log $y = kt + C$ ... (1)

In the year 1999, $t = 0$ and $y = 20000$.

Therefore, we get:

log 20000 = C ... (2)

In the year 2004, $t = 5$ and $y = 25000$.

Therefore, we get:

$$\log 25000 = k \cdot 5 + C$$

$$\Rightarrow \log 25000 = 5k + \log 20000$$

$$\Rightarrow 5k = \log\left(\frac{25000}{20000}\right) = \log\left(\frac{5}{4}\right)$$

$$\Rightarrow k = \frac{1}{5}\log\left(\frac{5}{4}\right) \qquad ...(3)$$

In the year 2009, $t = 10$ years.

$$\log y = 10 \times \frac{1}{5}\log\left(\frac{5}{4}\right) + \log(20000)$$

$$\Rightarrow \log y = \log\left[20000 \times \left(\frac{5}{4}\right)^2\right]$$

$$\Rightarrow y = 20000 \times \frac{5}{4} \times \frac{5}{4}$$

$$\Rightarrow y = 31250$$

Hence, the population of the village in 2009 will be 31250.

Q.17 If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and

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angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

Q.18 A man is known to speak truth 5 out of 6 times. He draws a ball from the bag containing 4 white and 6 black balls and reports that it is white. Find the probability that it is actually white? Do you think that speaking truth is always good? Ans : Required probability = 10/13. Speaking truth pays in a long run. Although sometimes lie told for a good cause is not bad.

Q.19 If $y = \sin^{-1}\left(\sqrt{x^4 - x^6} + \sqrt{x^2 - x^6}\right)$ Prove that $\frac{dy}{dx} = \frac{2x}{\sqrt{1 - x^4}} + \frac{1}{\sqrt{1 - x^2}}$.

Q.20 If $\sin^{-1}\frac{2p}{1+p^2} - \cos^{-1}\frac{1-q^2}{1+q^2} = \tan^{-1}\frac{2x}{1-x^2}$ then prove that $x = \frac{p-q}{1+pq}$.

Q.21 Evaluate : $\int_1^3 (5x^2 - e^x + 4)dx$ as a limit of sums Ans. $\frac{154}{3} - e^3 + e$

Q.22 Discuss the continuity and differentiability of f(x) = $\begin{cases} 1-x & x < 1 \\ (1-x)(2-x) & 1 \le x \le 2 \\ 3-x & x > 2 \end{cases}$. at x = 1 & x = 2 . f(x) is continous at x = 1 and discontinuous at x = 2 . F(x) is differentiable at x = 1 but f(x) is not continous at x = 2 there fore it is not differentiable at x = 2 .

PART – C

Q.23 Three shopkeepers A, B, C are using polythene, handmade bags (prepared by prisoners), and newspaper's envelope as carry bags. It is found that the shopkeepers A, B, C are using (20, 30, 40), (30, 40, 20), (40, 20, 30) polythene, handmade bags and newspapers envelopes respectively. The shopkeepers A, B, C spent Rs. 250, Rs. 270& Rs. 200 on these carry bags respectively. Find the cost of each carry bags using matrices. Keeping in the mind the social &

	environmental conditions, which shopkeeper is better? Why? Ans : Polythene = Rs.1, hand made bag=Rs.5, newspaper's envelope=Rs.2, shopkeeper A is better for environmental conditions. As he is using least number of polythene. Shopkeeper B is also better for social condition as he is using hand made bags (prepared by prisoners). Shopkeeper C is better to as the newspapers envelope used by him give employment to some people.
Q.24	Find the probability that a year chosen at random has 53 Sundays. ***Solution:*** Let P(L) be the probability that a year chosen at random is leap P(L) = 1/4, $\Rightarrow P(\bar{L})$ = ¾.Let P(S) be the probability that an year chosen at random has 53 Sundays . so that P(S) = P(L). P(S/L) + P(L').P(S/L').Now P(S/L) = probability that a leap year has 53 Sundays. **A leap year has 366 days, 52 weeks + 2 days; the remaining 2 days may be Sunday – Monday, M – T, T – W , W-Th, Th-F, F-Sat or Sat –S.** Out of the 7 possibilities, 2 are favourable. $\Rightarrow P(S/L) = \frac{2}{7}$. Similarly $P(S/\bar{L}) = \frac{1}{7} \Rightarrow P(S) = \frac{1}{4}.\frac{2}{7} + \frac{3}{4}.\frac{1}{7} = \frac{5}{28}$.
Q.25	Find the area of the origin : $\{(x,y): 0 \le y \le x^2, 0 \le y \le x+2; 0 \le x \le 3\}$

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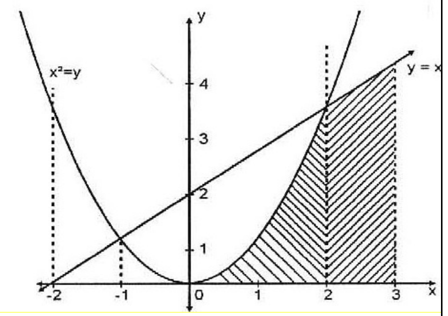
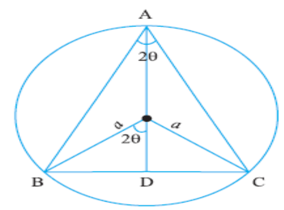
	For correct figure getting points of intersection as x=-1, x=2 Required area = $\int_0^2 x^2 dx + \int_2^3 (x+2)dx$ $\left[\frac{x^3}{3}\right]_0^2 + \left[\frac{(x+2)^2}{2}\right]_2^3$ $\frac{8}{3} + \frac{25}{2} - 8 = \frac{43}{6}$ sq.U  **OR** Find the ratio of the areas into which curve $y^2 = 6x$ divides the region bounded by $x^2 + y^2 = 16$. *ratiois* $(16\pi - 2\sqrt{3}):(4\pi + \sqrt{3})$
Q.26	An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius *a*. Show that the area of triangle is maximum when $\theta = \frac{\pi}{6}$. **Solution** Let ABC be an isosceles triangle inscribed in the circle with radius *a* such that AB = AC. AD = AO + OD = *a* + *a* cos2θ and BC = 2BD = 2*a* sin2θ .  Therefore, area of the triangle ABC i.e. $\Delta = \frac{1}{2}$BC . AD $= \frac{1}{2} 2a \sin 2\theta . (a + a\cos 2\theta) = a^2 \sin 2\theta (1 + \cos 2\theta) \Rightarrow \Delta = a^2 \sin 2\theta + \frac{1}{2} a^2 \sin 4\theta$ Therefore, $\frac{d\Delta}{d\theta} = 2a^2\cos 2\theta + 2a^2 \cos 4\theta = 2a^2(\cos 2\theta + \cos 4\theta)$ $\frac{d\Delta}{d\theta} = 0 \Rightarrow \cos 2\theta = -\cos 4\theta = \cos(\pi - 4\theta)$ $\frac{d\Delta}{d\theta} = 0 \Rightarrow \cos 2\theta = -\cos 4\theta = \cos(\pi - 4\theta)$

	Therefore, $2\theta = \pi - 4\theta \Rightarrow \theta = \frac{\pi}{6}$ $\frac{d^2\Delta}{d\theta^2} = 2a^2(-2\sin 2\theta - 4\sin 4\theta) < 0$ (at $\theta = \frac{\pi}{6}$). Therefore, Area of triangle is maximum when $\theta = \frac{\pi}{6}$. OR A point on the hypotenuse of a right triangle is at a distance 'a' and 'b' from the sides of the triangle. Show that the minimum length of the hypotenuse is $[a^{2/3} + b^{2/3}]^{3/2}$. Ans : $l = a\,\mathrm{cosec}\,\theta + b\sec\theta \therefore \tan\theta = \sqrt[3]{\frac{a}{b}} \Rightarrow \mathrm{cosec}\,\theta = \frac{(a^{2/3}+b^{2/3})^{1/2}}{a^{2/3}}$ & $\sec\theta = \frac{(a^{2/3}+b^{2/3})^{1/2}}{b^{2/3}}$
Q.27	Avinash has been given two lists of problems from his mathematics teacher with the instruction to submit not more than 100 of them correctly solved for marks. The problems in the first list are worth 10 marks each and those in the second list are worth 5 marks each. He knows from past experience that he requires on an average of 4 minutes to solve a problem of 10 marks and 2 minutes to solve a problem of 5 marks. He has other subjects to worry about; he cannot devote more than 4 hours to his mathematics assignment. With reference to manage his time in best possible way, how many problems from each list shall he do to maximize his marks? What is the importance of time management for students? Ans $x \ge 0; y \ge 0; x + y \le 100; 4x + 2y \le 240; z = 10x + 5y$ 20 problems from first list and 80 problems from second list. Student who divide time for each subject per day according to their need don't feel burden of any subject before the examination.
Q.28	Find the foot of the perpendicular from P(1, 2, 3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ .Also obtain the equation of the plane containing the line and the point (1, 2, 3). Foot of the perpendicular = (3,5,9)

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	the equation of plane :18x-22y+5z+11= 0
Q.29	Let X be a non – empty set. P(x) be its power set. Let '* be an operation defined on element of P(x) by A * B = A ∩ B ∀ A, B ∈ P(X) Then, (i) Prove that * is a binary operation in P(X). (ii) Is* commutative ? (iii) Is* associative? (iv)find the identity element in P(X) w.r.t * . (v) find the all the invertible element of P(X) (vi) if O is another binary operation defined on P(X) as A O B = A ∪ B then verify that O distribution itself over *. (iv) I is the identity element of p (x) $A*I = A = I*A \Rightarrow A\cap I = A = I \cap A$(iii) Let S be invertiable element of p (x) $A*S = X = S*A \Rightarrow A \cap S = X = S \cap A \Rightarrow A = S = X$ (V) AO(B *C)= $A\cup(B\cap C) = (A\cup B)\cap(A\cup C) = (AOB)*(AOC)$
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	A MAN WHO DOESN'T TRUST HIMSELF ; CAN NEVER TRULY TRUST ANYONE ELSE

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