Series SKS/1/C

कोड नं. **65/1/1**
Code No.

रोल नं.
Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **12** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **29** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **12** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 100*

Time allowed : 3 hours *Maximum Marks : 100*

SECTION – A

Q.1 Let f: R → R be defined by $f(x) = x^2 + 1$. Find the pre image of 17.

Q.2 Write the value of $\tan^{-1}\left\{\sin\left(\frac{-\pi}{2}\right)\right\}$

Q.3 If $X_{m \times 3} Y_{p \times 4} = Z_{2 \times b}$, for three matrices X, Y, Z, find the value of m, p and b.

Q.4 If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of a_{32}. A_{32}.

Q.5 If A is a square matrix of 3 x 3 order and $|A| = 5$, find the value of $|A \text{ adj} A|$

Q.6 Write the degree of the differential equation: $x^3\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$

Q.7 Find the angle between two vectors $\mathbf{a}$ and $\mathbf{b}$ with magnitude 1 and 2 respectively and when $|\mathbf{a} \times \mathbf{b}| = \sqrt{3}$

Q.8 If the position vector $\mathbf{a}$ of the point (-5, λ) be such that $|\mathbf{a}| = 13$, find λ.

Q.9 If the x- coordinate of a point P on the join of Q (2, 2, 1) and R(5, 1, -2) is 4, then find its z- coordinate.

Q.10 The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (marginal revenue). If the total revenue (in rupees) received from the sale of x units of a product is given by R(x) = 3x2 + 36x + 5, find the marginal revenue, when x = 5, and write which value does the question indicate?

SECTION – B

Q.11 If R_1 and R_2 are two equivalence relations in a given set A, show that $R_1 \cap R_2$ is also an equivalence relation.

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Q.12 If $\cos^{-1}\left(\frac{x}{2}\right)+\cos^{-1}\left(\frac{y}{3}\right)=\theta$, then prove that $9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$.

OR

Simplify: $\tan^{-1}\left(\frac{3\sin 2x}{5+3\cos 2x}\right)+\tan^{-1}\left(\frac{1}{4}\tan x\right)$

Q.13 Using properties of determinant prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

Q.14 Find the interval in which the function f given by $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

Q.15 The function f(x) is defined as $f(x)=\begin{cases} x^2+ax+b & 0\le x<2 \\ 3x+2 & 2\le x\le 4 \\ 2ax+5b & 4<x\le 8 \end{cases}$ if f(x) is continuous on [0, 8], find a and b.

OR

If $x = \sin(t)$, $y = \sin(pt)$, prove that $(1 - x^2)y_2 - xy_1 + p^2y = 0$.

Q.16 Differentiate $\cos^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ w.r.t. $\cos^{-1}x^2$

Q.17 Evaluate: $\int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

Q.18 Solve the differential equation: $y+\frac{d}{dx}(xy)=x(\sin x+\log x)$

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Q.19 Solve the differential equation: $x\cos\frac{y}{x}(ydx+xdy)=y\sin\frac{y}{x}(xdy-ydx)$

Q.20 The scalar product of a vector with vectors $3\hat{i}-5\hat{k}$, $2\hat{i}+7\hat{j}$ and $\hat{i}+\hat{j}+\hat{k}$ are respectively -1, 6 and 5. Find the vector.

Q.21 Find the equation of the line passing through the point P (4, 6, 2) and the point of intersection of the line $\frac{x-1}{3}=\frac{y}{2}=\frac{z+1}{7}$ and the plane x + y – z = 8.

OR

Find the equations of the line through the point (3, 0, 1) and parallel to the planes x + 2y = 0 and 3y – z = 0.

Q.22 In a self-assessment survey 60% persons claimed that they never indulged in corruption, 40% persons claimed that they always spoke the truth and 20% said that they neither indulged in corruption nor told lies. A person is selected at random out of this group.

(i) If the person never indulged in corruption, find the probability that she/he speaks the truth.

(ii) If the person always speaks the truth then find the probability that she/he claim to have never indulged in corruption.

(iii) What values have been discussed in this question?

SECTION – C

Q.23 Given that A = $\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$ find A^{-1}. Hence using A^{-1} solve the system of equations: x + 2y + z = 4, - x + y + z = 0, x - 3y + z = 2

Q.24 Show that the right circular cylinder of given volume open at the top has minimum total surface area, provided its height is equal to radius of its base.

OR

Find the condition for the curves $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $xy = c^2$ to intersect orthogonally.

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Q.25 Using integration, find the area of the triangle ABC with vertices as A(-1, 0), B(1, 3) and C(3, 2).

Q.26 Find the equation of plane passing through intersection of the planes $4x - y + z = 10$ and $x + y - z = 4$ and parallel to the line with D. R.'s <2, 1, 1>. Also find the perpendicular distance of the point (1, 1, 1) from this plane.

OR

Find the distance of the point (2, 3, 4) from the plane $3x + 2y + 2z + 5 = 0$ measured parallel to the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$

Q.27 A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produces 5% and 7% defective items respectively. A is on job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Q.28 Prove that $\int_0^{\pi/2} \log(\tan\theta + \cot\theta)d\theta = \pi \log 2.$

Q.29 A factory owner purchases two types of machines A and B for his factory to produce 'Eco-friendly' vehicles. The requirement and limitations for the machines are as follows:

	Area occupied by machine (in sq. m)	Labour force for each machine (man)	Daily Output in units
Machine A	1000	12	60
Machine B	1200	8	40

He has an area of 9,000 sq. m available and 72 skilled men who can operate the machines. How many machines of each type should he buy to maximise the daily output? Do you agree we should use 'Eco-friendly' vehicles for transportation?

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