Sample Paper – 2014

Class – XII

Subject – SUBJECT- MATHEMATICS

TIME:- 3 hrs **M.M.-100**

GENERAL INSTRUCTIONS:- 1. All questions are compulsory.

2. Section A, Q-1 to 10 carry 1mark each.

3. Section B, Q-11 to 22 carry 4 marks each.

4. Section C, Q-23 to 29 carry 6 marks each.

SECTION-A

1. If $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$, find $fog(x)$?
2. Find the value of $\cos^{-1}(\frac{-1}{2}) + 3\sin^{-1}\frac{1}{2}$
3. Let A be a square matrix of order 3x3 and k is a scalar then $|kA|$ = _____
4. If $\begin{vmatrix} 3 & x \\ 4 & 5 \end{vmatrix} = 3$ find the value of x?
5. Compute the product of $\begin{bmatrix} 2 & 3 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$

6. If x $=\sqrt{a^{\sin^{-1}t}}$, find dx/dt.

7. Evaluate: $\int \tan^2(2x-3)dx$

8. Find the value of x for which x($\hat{i}+\hat{j}+\hat{k}$) is a unit vector

9. Find a unit vector perpendicular to the vectors ($3\hat{i}+2\hat{j}+5\hat{k}$) and ($\hat{i}-3\hat{j}+\hat{k}$)?

10. Find the equation of the line which passes through the point (3,4,5) and is parallel to the vector $2\hat{i}+2\hat{j}+\hat{k}$?

Section B

11. Let A be the set of all real numbers except -1 i.e. A = R – {-1}. Let * be defined on A as a*b = a + b + ab for all a,b in A.

Prove that

I. * is a binary operation on A.

II. The given operation is commutative as well as associative.

III. The number 0 is the identity element.

IV. Every element of A has -a / (1+a) as its inverse.

OR

If $f: R \to R$ is defined by $f(x) = 2x+7$. Prove that f is a bijection. Also find inverse of f

12. Prove the following equation: $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

13. Prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

14. If the function f(x) = $\begin{cases} \frac{\sin(a+1)x + \sin x}{x} & for\ x < 0 \\ c & for\ x = 0 \\ \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}} & for\ x > 0 \end{cases}$ is continuous at x = 0, find the relation between a and b and c ?

15. If $\sin y = x\sin(a+y)$,prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

OR

If x = sin t and y = sin pt , prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

16. Separate $\left[0, \frac{\pi}{2}\right]$ into sub intervals in which f(x) = $\sin^4 x + \cos^4 x$ is strictly increasing or decreasing

OR

Find the equation of the tangent and normal to the curve $x = 1-\cos\theta,\ y = \theta - \sin\theta$ *at* $\theta = \frac{\pi}{4}$

17. Evaluate: $\int \frac{x^2+x+1}{(x^2+1)(x+2)}dx$.

18. Solve the differential equation: $x\frac{dy}{dx} + y = x\cos x + \sin x$, given $y = \frac{\pi}{2}$ when x =1.

19. Solve the differential equation : $x\cos\left(\frac{y}{x}\right)dy = \left(y\cos\left(\frac{y}{x}\right) + x\right)dx$

20.

Find the value of λ, if the points with position vectors $3\hat{i}-2\hat{j}-\hat{k}$, $2\hat{i}+3\hat{j}-4\hat{k}$, $-\hat{i}+\hat{j}+2\hat{k}$ and $4\hat{i}+5\hat{j}+\lambda\hat{k}$ are coplanar.

OR

If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them then prove that $\cos\frac{\theta}{2}=\frac{1}{2}|\vec{a}+b|$

21. Find the shortest distance between the lines whose vector equations are

$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and

$\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$

22.There is a group of 50 people who are patriotic out of which 20 believe in non violence. Two persons are selected at random out of them, write the probability distribution for the selected persons who are non-violent. Also find the mean and variance of the distribution. **Explain the importance of non-violence in patriotism.**

SECTION-C

23. Find the solution of following equations by matrix method : x-y+z= 4 , x-2y-2z = 9 ,2x+y+3z = 1 .

24. An open box with square base is to be made out of a given quantity of sheet of area a^2. Show that the maximum volume of the box is $a^3/6\sqrt{3}$

25. Find the area of the region enclosed between the circles $x^2+y^2=1$ and $(x-1)^2+y^2=1$.

(OR)

Find the area of the region {(X,Y): $0 \le y \le x^2, 0 \le y \le x+1, 0 \le x \le 2$ }

26 . Evaluate : $\int_0^a \sin^{-1}\sqrt{\frac{a}{a+x}}\,dx$

27. In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the prob. That he copies the answer is 1/6.

The probability that the answer is correct, given that he copied it, is 1/8. Find the probability that he knew the answer to the question, given that he answered it correctly.

Which values are ignored by the examinee if he copies the answer?

28. Find the distance of the point (1, -2, 3) from the plane x – y + z = 5, measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$

OR

Show that the lines $\frac{x-4}{1}=\frac{y-3}{4}=\frac{z-2}{5}$ and $\frac{x-3}{1}=\frac{y-2}{-4}=\frac{z+3}{5}$ intersect. Find the point of intersection and the equation of the plane containing them.

29**.**A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs 5760 and has space for at most 20 items. A electronic sewing machine costs him Rs 360 and a manually operated machine RS 240. He can sell an electronic sewing machine at a profit of Rs 22 and a manually operated machine for Rs 18. Assuming that he can sell all the items that he can buy how should he invest his money in order to maximize the profit**.** Form a LPP and solve it graphically. **Keeping the rural background in mind justify the 'value' to be promoted for the selection of the manually operated machine.**

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