TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

22 YEAR OF QUALITY EDUCATION

CODE:- AG-TS-8-3689 **REG.NO:-TMC -D/79/89/36**

General Instructions :-

1. All question are compulsory.
2. The question paper consists of 26 questions divided into three sections A,B and C. Section – A comprises of 6 question of 1 mark each. Section – B comprises of 13 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one If the alternatives in all such questions.
4. Use of calculator is not permitted.
5. Please check that this question paper contains 6 printed pages.
6. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

CLASS – XII ***CBSE*** ***MATHEMATICS***

Time : 3 Hours Maximum Marks : 100

PRE-BOARD EXAMINATION 2013 -14

PART – A

Q.1 Prove that : $\sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)=\frac{14}{15}$. Ans

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$2\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}\left(\frac{3}{4}\right)=\sin^{-1}\left(\frac{3}{5}\right)$ & $\tan^{-1}\left(2\sqrt{2}\right)=\cos^{-1}\left(\frac{1}{3}\right)$ $\frac{3}{5}+\frac{1}{3}=\frac{14}{15}$

Q.2 If $\int_0^1 (3x^2+2x+k)dx=0$, find the value of k. Ans.k = -2

Q.3 If $A^T=\begin{bmatrix}-2 & 3\\ 1 & 2\end{bmatrix}$ & $B=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$ then find $(A+2B)^T$. ANS : $\begin{bmatrix}-4 & 5\\ 1 & 6\end{bmatrix}$

Q.4 If the binary operation * on the set of integers Z, is defined by $a * b = a+3b^2$, then find the value of 2 * 4. {Ans.50

Q.5 If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$. then find the angle between $\vec{a}$ and $\vec{b}$. Ans $\frac{\pi}{2}$.

Q.6 Find the value of λ , so that the lines $\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other {Ans. $\lambda=7$

Q.7 Evaluate $\int\frac{dx}{x\cos^2(1+\log x)}$. Ans $I=\tan(1+\log x)+c$.

Q.8 A matrix X has $(a+b)$ rows and $(a+2)$ columns while the matrix Y has (b+1) rows and (a+3) columns. Both the matrices XY and YX exist. Find the values of a and b . ANS : a = 2 & b = 3

Q.9 From the point A (2, 3, 8) in space, a shooter aims to hit the target at P (6, 5, 11). If the line of fire $\frac{x-2}{4}=\frac{y-3}{2}=\frac{z-8}{3}$, what do you think about the success of the shooter? ans : Since both the points lie on the

	line of fire so, the shooter will be successful in his attempt.
Q.10	Find the angle between two vectors $\vec{a}\,\&\,\vec{b}$ having the same length $\sqrt{2}$ and their scalar product is -1. Ans $\vec{a}$ and $\vec{b} = \frac{2\pi}{3}$
	PART – B
Q.11	Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) + 3 = 0$ Ans $-3\hat{i} + 5\hat{j} + 2\hat{k}$
Q.12	Evaluate $\int \frac{x^2 + x + 1}{(x+2)(x^2+1)}$. Ans $\frac{3}{5}\log\lvert x+2\rvert + \frac{1}{5}\left[\log\lvert x^2+1\rvert + \tan^{-1}x\right] + C$
Q.13	Show that $\frac{1}{2}\vec{AC} \times \vec{BD}$ represents the vector area of the plane quadrilateral ABCD. Also find the area of quadrilateral whose diagonals are $4i - j - 3k\ \&\ -2i + j - 2k$. Ans. $\frac{15}{2}unit^2$
Q.14	Is f (x) = $\lvert x-1\rvert + \lvert x-2\rvert$ continuous and differentiable at x = 1,2 . Ans : f (x) is continous at x 1 ; 2 but not differentiable at x= 1 & 2 .
Q.15	Obtain a differential equation of the family of circles touching the x-axis at origin. Ans: Equation of circle : $x^2 + (y-a)^2 = a^2$ Rrquired differential eqn $\left(x^2 - y^2\right)y_1 = 2xy$
Q.16	Using properties of determinants, prove that :

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	$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = \left(1 + a^2 + b^2 + c^2\right).$
Q.17	Find the particular solution, satisfying the given condition, for the following differential equation . $\frac{dy}{dx} - \frac{y}{x} + \cos ec\left(\frac{y}{x}\right) = 0,\ y = 0\ when\ x = 1$ Ans : $\log\lvert x\rvert + \log e = \cos\frac{y}{x} \Rightarrow \log\lvert ex\rvert = \cos\frac{y}{x}$ **OR** Solve the differential equation $\left[\frac{e^{-2\sqrt{y}}}{\sqrt{y}} - \frac{x}{\sqrt{y}}\right]\frac{dy}{dx} = 1$; $(y \neq 0)$ and $y(1) = 2$. ANS : Ans

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$$\left[\frac{e^{-2\sqrt{y}}}{\sqrt{y}}-\frac{x}{\sqrt{y}}\right]\frac{dy}{dx}=1\ ;\ (y\neq 0)$$

$$\frac{dx}{dy}+\frac{x}{\sqrt{y}}=\frac{e^{-2\sqrt{y}}}{\sqrt{y}}$$

$$IF=e^{\int\frac{1}{\sqrt{y}}dy}=e^{2\sqrt{y}}$$

solution of given differential equation is given by

$$x.IF=\int IF.Q\,dy+c$$

$$xe^{2\sqrt{y}}=\int\frac{e^{-2\sqrt{y}}}{\sqrt{y}}e^{2\sqrt{y}}dy+c$$

$$xe^{2\sqrt{y}}=\int\frac{1}{\sqrt{y}}dy+c$$

$$xe^{2\sqrt{y}}=2\sqrt{y}+c$$

Now if $y(1)=2$. Then c = $e^{2\sqrt{2}}-2\sqrt{2}$

$$xe^{2\sqrt{y}}=2\sqrt{y}+e^{2\sqrt{2}}-2\sqrt{2}$$

Q.18 let R_+ be the set of all non-negative real numbers Let f : $R_+\rightarrow[4,\infty):f(x)=x^2+4$. Show that f is invertible that find f^{-1}

Ans $f^{-1}(y)=\sqrt{y-4}$.

Q.19 The radius of a spherical diamond is measured as 7 cm with an error of 0.04 cm. Find the approximate error in calculating its volume. If the cost of 1 cm^3 diamond is Rs. 1000, what is the loss to the buyer of the diamond? What lesson do you get from this observation? ans : Approximate error in volume = 24.64 cm^3. Loss to the buyer = (Error in volume). (Cost) = Rs. 24,640. **Lesson :** A small error of 0.04 cm can result in huge loss of Rs. 24,640. So it is needed to be careful while . Taking measurements .

OR

Find the equation of the normal's to the curve y = x^3+2x+6 which are parallel to the line x + 14y + 4 = 0. Why are the fruits good for health? **Importance of fruits:** Fruits contain nutrients and vitamins which help our body in its proper growth and maintenance. Ans Equation of normal at (2,18) is x + 14y + 86 =0 & Equation of normal at (-2,-6) is x + 14y -254 = 0

Q.20 A football match may be either won , drawn or lost by the host country's team . So there are three ways of forecasting the result of any match , one correct and two incorrect . Find the probability forecasting at least three correct result for four matches .**Ans: p = 1 / 3 ; q = 2 / 3 ; n = 4 ;Required probability** $= p(x=3)+p(x=4)=4\bullet\frac{2}{3}\bullet\frac{1}{27}+\frac{1}{81}=\frac{1}{9}$

Q.21 If $x=a(\cos\theta+\log\tan\frac{\theta}{2})$ & $y=a\sin\theta$, find the value of $\frac{d^2y}{d\theta^2}$ & $\frac{d^2y}{dx^2}$ $at\ \theta=\frac{\pi}{4}$. Ans $\left(\frac{d^2y}{d\theta^2}\right)=-a\sin\theta=\frac{-a}{\sqrt{2}}$; $\left(\frac{d^2y}{dx^2}\right)=\frac{\tan\theta}{a\cos^3\theta}=\frac{2\sqrt{2}}{a}$

OR

Differentiate w.r.t.x: $y=\frac{(2x+3)\sqrt{3x-4}}{(x^2+1)^3}$, *find* $\frac{dy}{dx}$ Ans

$$\frac{dy}{dx}=\frac{(2x+3)\sqrt{3x-4}}{(x^2+1)^3}\left[\frac{2}{2x+3}+\frac{3}{2(3x-4)}-\frac{6x}{(x^2+1)}\right]$$

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Q.22 Prove that : $2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right]=\cos^{-1}\left(\frac{b+a\cos\theta}{a+b\cos\theta}\right)$.

OR

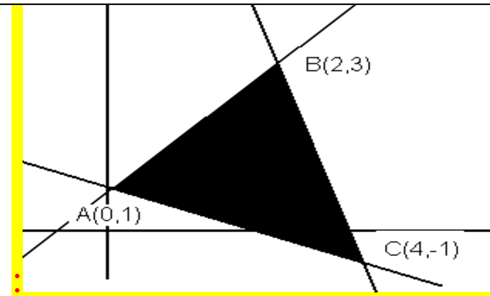
If $y=\cot^{-1}(\sqrt{\cos x})-\tan^{-1}(\sqrt{\cos x})$ Prove that $\sin y=\tan^2\frac{x}{2}$.

PART – C

Q.23 If $A=\begin{bmatrix}2 & 3 & 4\\ 5 & 4 & -6\\ 3 & -2 & -2\end{bmatrix}$ and $B=\begin{bmatrix}20 & 2 & 34\\ 8 & 16 & -32\\ 22 & -13 & 7\end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equation : $\frac{2}{x}+\frac{3}{y}+\frac{4}{z}=-3;;\frac{5}{x}+\frac{4}{y}-\frac{6}{z}=4;\frac{3}{x}-\frac{2}{y}-\frac{2}{z}=6$. Ans $\begin{bmatrix}1\\ -1\\ -2\end{bmatrix}$

Q.24 A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, find the number of rings and chains that should be manufactured per day , so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically. {Ans z = 300 x + 190 y $x+y\le 24$; $x+\frac{1}{2}y\le 16$; $x, y\ge 0$ Z is maximum at B (8,16) i.e., x = 8, y =16. Hence 8 gold ring and 16 chains must be produced per day to get a maximum profit of Rs 5,440

Q.25 Using integration, find the area of the triangle bounded by the lines x + 2y = 2, y – x = 1 and 2x + y = 7. Ans



$A_1=\int_{-1}^{3}\frac{7-y}{2}dy; A_2=\int_{1}^{3}(1+y)dy; A_3=\int_{-1}^{1}(2-2y)dy \Rightarrow A_1-A_2-A_3=6unit^2$

OR

Find the area bounded by the curve $y^2=4a^2(x-1)$ and the lines x = 1 and y = 4a. Ans $\int_0^{4a} x\,dy - \int_0^{4a} 1\,.dy = \frac{16a}{3}\ sq\ .unit$.

Q.26 In a school the school management committee wants to built a hall in the school . During the meeting it was decided that there should be 8 windows of same size in the hall for proper light and fresh air . Each window be in the form of rectangle surmounted by equilateral triangle . The total perimeter of each window is 15 meter . Find the dimensions of the rectangular part of each window so as to admit maximum light and fresh air through the whole opening . Write the two value points behind the decision . ans ; p = 3x + 2y = 15 where $y=\frac{15-3x}{2}$ $\therefore A=xy+\frac{\sqrt{3}x^2}{4}\Rightarrow\frac{1}{2}\left[15x-3x^2\right]+\frac{\sqrt{3}}{4}x^2$ $\therefore x=\frac{15}{6-\sqrt{3}}; y=\frac{15}{2}\left(\frac{3-\sqrt{3}}{6-\sqrt{3}}\right)m$. The two value points are : (i) fresh air and sun light is necessary for good helth of children . (ii) Large sized windows allow more light during day time so electrcity will be saved .

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Q.27	Evaluate $\int_0^2 (3^x + 5x^2 + 7x - 2)\,dx$ as limit of sums. Ans : $\frac{8}{\log 3} + \frac{70}{3}$
Q.28	In a remote area there is only one boys college . The post of principal is vacant and it is to be filled . there are three probable candidates i.e. A , B & C for the post of principal . The chances of their selection are in the proportion 4 : 2 : 3 respectively . The probability that A , if selected will introduce co-education in that college is .3 , the probability of B and c doing the same are .5 and .8 respectively . What is the probability that there will be co-education in the college ? Find the probability that B introduced co-education in college . Which value will be developed among the people of that remote area ? ans : P(E1)= 4/9 ; P9E20= 2/9 ;P(E3) = 3/9 & A= The event to introduce co-education in the college P(A/E1) = . 3 ; P(A/E2) = .5 & P(A/E3) = .8 . The probability that there will be co-education in the college = Using the law of total probability = P(E1)P(A/E1) + P(E2)P(A/E2) + P(E3)P(A/E3) = 46/90 = 23/45 . The probability that B introduced co-education in college = P(E2/A) = P(E2)P(A/E2)/ P(E1)P(A/E1) + P(E2)P(A/E2) + P(E3)P(A/E3) = 5/23. The value is reflected here that the girl/women should be educated at college level . If ladies will be highly educated , reflection can be seen in the families . **OR** There are three coins. One is a biased coin that comes up with tail 60% of the times, the second is also a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it showed heads. What is the probability that it was the unbiased coin? If shopkeeper provides polythene carry bag or cotton carry bags , which type of bag would you like to carry your items and why ? Ans : Let E1:selectionof first (biased) coin ;E2: selection of second (biased) coin ;E3: selection of third (unbiased) coin $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ Let A denote the event of getting a head Therefore, $P\left(\frac{A}{E_1}\right) = \frac{40}{100}$, $P\left(\frac{A}{E_2}\right) = \frac{75}{100}$, $P\left(\frac{A}{E_3}\right) = \frac{1}{2}$ $P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$ $= \frac{\frac{1}{3}\cdot\frac{1}{2}}{\frac{1}{3}\cdot\frac{40}{100} + \frac{1}{3}\cdot\frac{75}{100} + \frac{1}{3}\cdot\frac{1}{2}} = \frac{10}{33}$ ans ; We would like cotton carry bags because polythene carry bags contain toxic chemicals which are very harmful for helth . Also polythene is non-biodegradable so produces pollution in the environment .
Q.29	Find the equation of the plane through the intersection of planes 3x - y + 4 z = 0 and x + 3 y + 6 = 0 , whose perpendicular distance from the origin equal to 1 . Ans: Equation of plane: - x + 2y – 2z + 3 = 0 ; 2 x + y + 2 z +3 = 0
	UNLESS YOU BELIEVE, YOU WILL NOT UNDERSTAND.